LECTURE 4. COMPOSITION OF CONCURRENT FORCES

1. Purpose:

To study the graphical and analytical methods used in the composition and resolution of forces and to examine the conditions for the equilibrium of a particle under the influence of coplanar forces.

2. Equipment

- 1 Force table
- 1 level ring which serves as a “particle”
- 4 weight hangers
- 3 pulleys with quick-release clamps
- 1 set of slotted weights
- string
- 1 ruler

3. Background

Theory:

Scalars are quantities, which have magnitude only, such as 250, $56.35, and 10 gallons of gas. A number of physical quantities require more than that for their complete description. One type of such quantity, used quite extensively in physics, is called a vector for which both a magnitude and a direction are required for its complete specification. Can you think of any?

The handwritten notation, which distinguishes a vector quantity from a scalar, is an arrow above the letter representing the vector, thus $\vec{A}$. Printed materials (books, journals, etc.) often make use of boldface type. In the discussion to follow, the notation $\vec{A}$ will be used.

A. Vector Addition and Resolution

In many instances, a vector may be slid along its line of action and, for purposes of adding vectors, even translated parallel to itself without changing its effect as long as both magnitude and direction are unchanged. This fact is the basis for the geometric approach to the addition of vectors. Thus if we have $\vec{R} = \vec{A} + \vec{B}$ we place the tail of vector $\vec{B}$ at the head of vector $\vec{A}$. Vector $\vec{R}$ which is the vector sum of *A* and *B* is the vector of proper length (magnitude) and direction such that its tail coincides with that of $\vec{A}$ and its head coincides with that of $\vec{B}$. This method is known as the **Triangle Method** of vector addition. Note that for an analytical treatment of this problem the law of sines and the law of cosines are useful. What are they?

Another geometrical method for adding $\vec{A}$ and $\vec{B}$ is the so-called **Parallelogram Method**. Here the tails of $\vec{A}$ and $\vec{B}$ are brought together at the same point “O”. At the tip of $\vec{A}$ a line is drawn parallel to $\vec{B}$, and at the tip of $\vec{B}$ a line is drawn parallel to $\vec{A}$. The

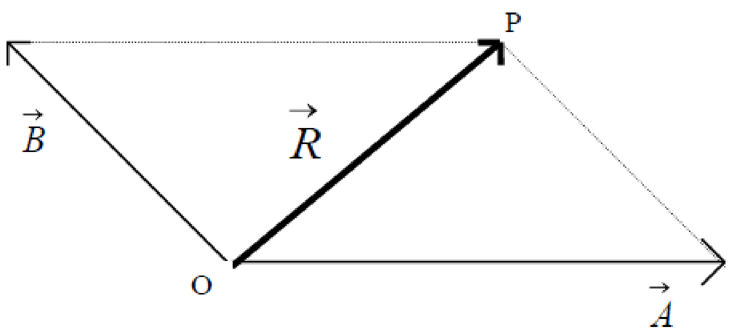


Figure 1

vector drawn from “O” to “P”, the point of intersection of these two lines, with head at “P” is the resultant vector $\vec{R}$

Equilibrium of Concurrent Forces (Force Table)

Vector addition is basically a process in which several vectors, acting together, are replaced by a single vector, which by itself has the same effect. Often it proves to be useful to go the other way, that is, to replace a single vector by several vectors which, acting together, have the same effect. It is not necessary that the new vectors need to be perpendicular to each other (orthogonal), but it often proves most convenient to arrange them so. In a plane, the given vector is replaced by its x- and y-components. Such a process is known as resolving the vector into its components. The components of vector *C* have the following magnitude (Note that the magnitude of $\vec{C}$ is given by C without the arrow).

$$C_x = C\cos\theta$$

$$C_y = C\sin\theta$$

Where θ is the angle between $\vec{C}$ and the x-axis.

The student should show that $C^2 = C_x{}^2 + C_y{}^2$

Vector addition in terms of components is especially simple. Thus if;

$$\vec{D} = \vec{A} + \vec{B} + \vec{C}$$

Then the magnitudes of the components of $\vec{D}$ are such that

$$D_x = A_x + B_x + C_x$$

$$D_y = A_y + B_y + C_y$$

$$D_z = A_z + B_z + C_z$$

Since each of these last three equations represents an algebraic expression, and since algebraic summations obey the commutative and associative laws, the corresponding vector addition also has the same commutation and association properties. The effect of multiplying a vector by a scalar is to change the magnitude, but not the direction.

We will define the equilibrium of a particle to be a state in which it undergoes no acceleration ($\sum\vec{F} = 0$). The condition that a particle is in equilibrium is that the vector sum of all the forces acting on it is zero.

Forces obey all the requirements laid down above for vectors, and so forces are vector quantities. In particular, if a particle is in equilibrium under the influence of several forces, then the forces must add up to zero. Another way of looking at it would be to say that any one of the forces is the equilibrant of the others. (The equilibrant of a set of forces is equal and opposite to their resultant.) Forces are useful for a laboratory experiment because of the ease with which they may be set up and controlled.

For our purposes, the “particle” upon which the forces act is a ring which is placed at the center of a horizontal table known as a force table. The force table should be carefully leveled at the outset. The ring is held in place with a pin while the forces are being applied and balanced. Forces are applied to the ring by means of light cords that run over pulleys placed around the rim of the table. The angular positions of the pulleys may be obtained from a scale around the rim of the table. The free ends of the cords have loops on which weight holders may be hung.

B. Laboratory Exercise

The experiment consists of balancing the ring at the center of the table under the action of unequal forces. Choose forces such that angles of 90° between forces are avoided as well as equal angles of 120°. Some experience is needed on the part of the observer to achieve equilibrium conditions with a minimum of frictional forces. One may pull the ring aside and observe its return to the center of the table. Again, when the ring seems to be correctly located, a vertical displacement and sudden release (snapping the ring) may help in determining the ultimate balance. Be sure to observe the following precautions.

1. Take care that the strings run true over the pulleys, i.e. that the portion of the string that passes over the table is horizontal and in the plane containing the pulley groove.

2. The strings should be radial for this equilibrium of a particle experiment.

3. Do not let the hook of a weight hanger touch a pulley.

4. Be sure to include the weights of the weight hanger as a part of the forces. In fact, it is good practice to list the weight of the hanger and each weight separately for individual applied forces.

4. Procedure:

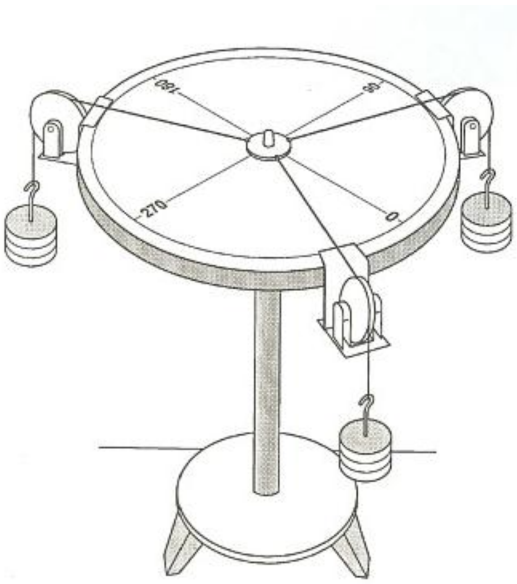
1. Select three forces, say A, B, and C, acting at odd angles. For convenience take one force at 0°. Find an equilibrium arrangement, experimentally.

Caution:

(1) The directions of the strings should intersect at the center of the ring.

(2) Do not load any string with more than 600gm, including the hanger.

2. Use a new group of four forces, say A, B, C, D, and obtain equilibrium experimentally. In this case we shall test directly the conditions of equilibrium.



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REPORTS

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1. **Purpose::………………………………………………………………………………………**
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2. **Results:**

Force = mass x 9,81

Table 1: Calculation

	Magnitude	Direction (angle)	x-component	y-component
Force **A**				
Force **B**				
Force **C = -(A**+**B**)				
Net component = **A**+**B**+**C**				

Table 2: Experiment

	Magnitude	Direction (angle)	x-component	y-component
Force **A**				
Force **B**				
Force **X**=**A**+**B**				
Force				

C = -(A+**B)**				
Net component = **A**+**B**+**C**				

Error:

$$\delta = \frac{\left|C_{cal.} - C_{\exp.}\right|}{C_{cal.}} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Table 3: Calculation

	Magnitude	Direction (angle)	x-component	y-component
Force **A**				
Force **B**				
Force **X**=**A**+**B**				
Force **C**				
Force **Y = A**+**B**+**C**				
Force **D**= **-(A**+**B**+**C)**				
Net component=**A**+**B**+**C**+**D**				

Table 4: Experiment

	Magnitude	Direction (angle)	x-component	y-component
Force **A**				
Force **B**				
Force **X**=**A**+**B**				
Force **C**				
Force **Y = A**+**B**+**C**				
Force **D**= **-(A**+**B**+**C)**				
Net component=**A**+**B**+**C**+**D**				

Error:

$$\delta = \frac{|D_{cal.} - D_{\exp.}|}{D_{cal.}} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

1. Discussion of results

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