

ENGLISH

For MATHEMATICS

CONTENTS

UNIT	TEXT	GRAMMAR	Page
	Preface.....	5
UNIT 1	– The Internet distance education – My future profession – Arithmetic operations	– Present Simple and Present Continuous	7
UNIT 2	– The history of personal computing – What is mathematics? – Fermat’s last theorem	– Present Simple – Past Simple	13
UNIT 3	– Fractions – J.E.Freund’s System of Natural Numbers Postulates	– The Present Perfect	20
UNIT 4	– Something about mathematical sentences – Inequalities – Mathematical signs and symbols	– Degrees of comparison	27
UNIT 5	– Thinking and reasoning in maths – Points and lines – How to find a website for information you need	– ING ending forms	34
UNIT 6	– Some advices to buying a computer – The Pythagorean property – Drawing a circle	– Modal verbs	40
UNIT 7	– Mathematical logic – The coordinate plane	– Infinitive after adjectives – Infinitive of purpose	47
UNIT 8	– Ratio and Proportion – History of the terms “ellipse”, “hyperbola” and “parabola” – Algorithms	– Past Participle – The Passive	55
UNIT 9	– What is an electronic computer? – Probability of occurrence	– Relative clauses	62
UNIT 10	– Sequences obtained by repeated multiplication. – Topology	– Conditionals – First and Zero – Some cases of irregular plural nouns.	69
	– Unending progressions		

UNIT 11	– Mappings – Why learn mathematics?	– Second conditionals	77
UNIT 12	– Multimedia – Matrices – William Rowan Hamilton	– ing / –ed participle clauses – Some cases of irregular plural nouns (continued)	85
UNIT 13	– Mathematics and modern civilization – The derivative of a function and some applications of the derivative	– Past Perfect Simple and Continuous – Adverbs	93
UNIT 14	– Thinking about the use of virtual reality in computer war games – Zeno’s paradoxes – George Cantor	– Reported speech – Some cases of irregular plural nouns (continued)	102
	References.....	161

PREFACE

This course is intended for students of non-English major in the Department of Mathematics, Ho Chi Minh City University of Pedagogy.

The course aims at developing students' language skills in an English context of mathematics with emphasis on reading, listening, speaking and writing. The language content, mainly focuses on: firstly, key points of grammar and key functions appropriate to this level; secondly, language items important for decoding texts mathematical; thirdly, language skills developed as outlined below.

This textbook contains 14 units with a Glossary of mathematical terms and a Glossary of computing terms and abbreviations designed to provide a minimum of 150 hours of learning.

Course structural organization:

Each unit consist of the following components:

PRESENTATION: The target language is shown in a natural context.

- ***Grammar question:***

Students are guided to an understanding of the target language, and directed to mastering rules for their own benefit.

PRACTICE:

Speaking, listening, reading and writing skills as well as grammar exercises are provided to consolidate the target language.

SKILLS DEVELOPMENT:

Language is used for realistic purposes. The target language of the unit reappears in a broader context.

- ***Reading and speaking:***

At least one reading text per unit is intergrated with various free speaking activities.

- ***Listening and speaking:***

At least one listening activity per unit is also intergrated with free speaking activities.

- ***Writing:***

Suggestions are supplied for writing activities per unit.

- ***Vocabulary:***

At least one vocabulary exercise per unit is available.

TRANSLATION:

The translation will encourage students to review their performance and to decide which are the priorities for their own future self-study.

Acknowledgements:

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Our special thanks are extended to the colleagues, who have done with their critical response and particular comments. Also, we would like to thank all those student–mathematicians who supplied all the necessary mathematical material to help us write this textbook.

UNIT 1

PRESENT SIMPLE & PRESENT CONTINUOUS

PRESENTATION

1. *Read the passage below. Use a dictionary to check vocabulary where necessary.*

INTERNET DISTANCE EDUCATION

The World Wide Web (www) is beginning to see and to develop activity in this regard and this activity is increasing dramatically every year. The Internet offers full university level courses to all registered students, complete with real time seminars and exams and professors' visiting hours. The Web is extremely flexible and its distance presentations and capabilities are always up to date. The students can get the text, audio and video of whatever subject they wish to have.

The possibilities for education on the Web are amazing. Many college and university classes presently create web pages for semester class projects. Research papers on many different topics are also available. Even primary school pupils are using the Web to access information and pass along news to others pupils. Exchange students can communicate with their classmates long before they actually arrive at the new school.

There are resources on the Internet designed to help teachers become better teachers – even when they cannot offer their students the benefits of an on-line community. Teachers can use university or college computer systems or home computers and individual Internet accounts to educate themselves and then bring the benefits of the Internet to their students by proxy.

2. *Compare the sentences below.*

- a. "This activity increases dramatically every year".
- b. "Even primary school pupils are using the Web to access information".

3. *Grammar questions*

- a. Which sentence expresses a true fact?
- b. Which sentence expresses an activity happening now or around now?

◆ **Note**

Can is often used to express one's ability, possibility and permission. It is followed by an infinitive (without *to*).

Read the passage again and answer the questions.

- a. What can students get from the Web?
- b. How can Internet help teachers become better teachers?

PRACTICE

1. Grammar

1.1 Put the verb in brackets into the correct verb form (the Present Simple or the Present Continuous) and then solve the problem.

Imagine you (wait) at the bus stop for a friend to get off a bus from the north. Three buses from the north and four buses from the south (arrive) about the same time. What (be) the probability that your friend will get off the first bus? Will the first bus come / be from the north?

1.2 Complete these sentences by putting the verb in brackets into the Present Simple or the Present Continuous.

- a. To solve the problem of gravitation, scientists (consider) time-space geometry in a new way nowadays.
- b. Quantum rules (obey) in any system.
- c. We (use) Active Server for this project because it (be) Web-based.
- d. Scientists (trace and locate) the subtle penetration of quantum effects into a completely classical domain.
- e. Commonly we (use) C++ and JavaScript.
- f. At the moment we (develop) a Web-based project.
- g. Its domain (begin) in the nucleus and (extend) to the solar system.
- h. Right now I (try) to learn how to use Active Server properly.

1.3 Put "can", "can not", "could", "could not" into the following sentences.

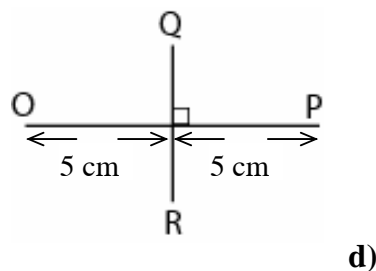
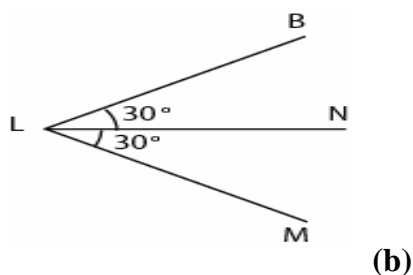
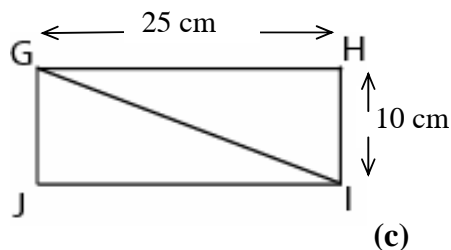
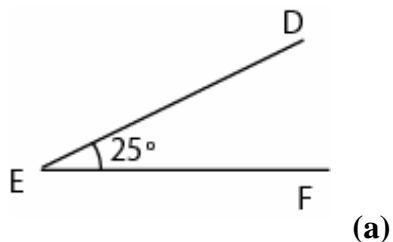
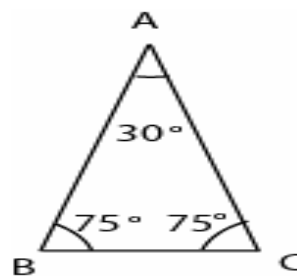
- a. Parents are finding that they no longer help their children with their arithmetic homework.
- b. The solution for the construction problems be found by pure reason.
- c. The Greeks solve the problem not because they were not clever enough, but because the problem is insoluble under the specified conditions.
- d. Using only a straight-edge and a compass the Greeks easily divide any line segment into any number of equal parts.
- e. Web pages..... offer access to a world of information about and exchange with other cultures and communities and experts in every field.

2. Speaking and listening

2.1 Work in pairs

Describe these angles and figures as fully as possible.

Example: ABC is an isosceles triangle which has one angle of 30° and two angles of 75° .



2.2 How are these values spoken?

a) x^2

d) x^{n-1}

g) $\sqrt[3]{x}$

b) x^3

e) x^{-n}

h) $\sqrt[n]{x}$

c) x^n

f) \sqrt{x}

i) $\sqrt[3]{(x-a)^2}$

SKILLS DEVELOPMENT

• Reading

1. Pre – reading task

1.1 Do you know the word “algebra”?

Do you know the adjective of the noun “algebra”?

Can you name a new division of algebra?

1.2 Answer following questions.

- What is your favourite field in modern maths?
- Why do you like studying maths?

2. Read the text.

MY FUTURE PROFESSION

When a person leaves high school, he understands that the time to choose his future profession has come. It is not easy to make the right choice of future profession and job at once. Leaving school is the beginning of independent life and the start of a more serious examination of one's abilities and character. As a result, it is difficult for many school leavers to give a definite and right answer straight away.

This year, I have managed to cope with and successfully passed the entrance exam and now I am a "freshman" at Moscow Lomonosov University's Mathematics and Mechanics Department, world-famous for its high reputation and image.

I have always been interested in maths. In high school my favourite subject was *Algebra*. I was very fond of solving algebraic equations, but this was elementary school algebra. This is not the case with university algebra. To begin with, *Algebra is a multifold subject*. Modern abstract deals not only with equations and simple problems, but with *algebraic structures* such as "groups", "fields", "rings", etc; but also comprises new divisions of algebra, e.g., *linear algebra, Lie group, Boolean algebra, homological algebra, vector algebra, matrix algebra* and many more. Now I am a first term student and I am studying the fundamentals of calculus.

I haven't made up my mind yet which field of maths to specialize in. I'm going to make my final decision when I am in my fifth year busy with my research diploma project and after consulting with my scientific supervisor.

At present, I would like to be a maths teacher. To my mind, it is a very noble profession. It is very difficult to become a good maths teacher. Undoubtedly, you should know the subject you teach perfectly, you should be well-educated and broad minded. An ignorant teacher teaches ignorance, a fearful teacher teaches fear, a bored teacher teaches boredom. But a good teacher develops in his students the burning desire to master all branches of modern maths, its essence, influence, wide-range and beauty. All our department graduates are sure to get jobs they would like to have. I hope the same will hold true for me.

Comprehension check

1. Are these sentences True (T) or False (F)? Correct the false sentences.

- a. The author has successfully passed an entrance exam to enter the Mathematics and Mechanics Department of Moscow Lomonosov University.
- b. He liked all the subjects of maths when he was at high school.
- c. Maths studied at university seems new for him.
- d. This year he's going to choose a field of maths to specialize in.
- e. He has a highly valued teaching career.
- f. A good teacher of maths will bring to students a strong desire to study maths.

2. Complete the sentences below.

- a. To enter a college or university and become a student you have to pass.....
- b. Students are going to write theirin the final year at university.
- c. University students show their essays to their.....

3. Work in groups

a. *Look at the words and phrases expressing personal qualities.*

- | | |
|----------------------|---------------------------|
| – sense of humour | – good knowledge of maths |
| – sense of adventure | – children – loving |
| – patience | – intelligence |
| – reliability | – good teaching method |
| – kindness | – interest in maths |

b. *Discussion*

What qualities do you need to become a good maths teacher?

c. *Answer the following questions.*

- c.1. Why should everyone study maths? What about others people?
- c.2. University maths departments have been training experts in maths and people take it for granted, don't they?
- c.3. When do freshmen come across some difficulties in their studies?
- c.4. How do mathematicians assess math studies?

• Listening

1. Pre – listening

All the words below are used to name parts of computers. *Look at the glossary to check the meaning.*

mainframe – mouse – icon – operating system – software – hardware – microchip

2. *Listen to the tape. Write a word next to each definition.*

- a. The set of software that controls a computer system.....
- b. A very small piece of silicon carrying a complex electrical circuit.....
- c. A big computer system used for large - scale operations.
- d. The physical portion of a computer system.
- e. A visual symbol used in a menu instead of natural language.
- f. A device moved by hand to indicate positions on the screen.....
- g. Data, programs, etc., not forming part of a computer, but used when operating it
.....

TRANSLATION

Translate into Vietnamese.

Arithmetic operations

1. **Addition:** The concept of adding stems from such fundamental facts that it does not require a definition and cannot be defined in formal fashion. We can use synonymous expressions, if we so much desire, like saying it is the process of combining.
Notation: $8 + 3 = 11$; 8 and 3 are the *addends*, 11 is the *sum*.
2. **Subtraction:** When one number is subtracted from another the result is called the *difference* or *remainder*. The number subtracted is termed the *subtrahend*, and the number from which the subtrahend is subtracted is called *minuend*.
Notation: $15 - 7 = 8$; 15 is the subtrahend, 7 is the minuend and 8 is the remainder. Subtraction may be checked by addition: $8 + 7 = 15$.
3. **Multiplication:** is the process of taking one number (called the *multiplicand*) a given number of times (this is the *multiplier*, which tells us how many times the multiplicand is to be taken). The result is called the *product*. The numbers multiplied together are called the factors of the products.
Notation: $12 \times 5 = 60$ or $12.5 = 60$; 12 is the multiplicand, 5 is the multiplier and 60 is the product (here, 12 and 5 are the factors of product).
4. **Division:** is the process of finding one of two factors from the product and the other factor. It is the process of determining how many times one number is contained in another. The number divided by another is called the *dividend*. The number divided into the dividend is called the *divisor*, and the answer obtained by division is called the *quotient*.
Notation: $48 : 6 = 8$; 48 is the dividend, 6 is the divisor and 8 is the quotient. Division may be checked by multiplication.

UNIT 2

PAST SIMPLE

PRESENTATION

1. *Here are the past tense forms of some verbs. Write them in the base forms.*

..... took decided
..... believed set
..... was (were) went
..... reversed made

Three of them end in *-ed*. They are the past tense form of regular verbs. The others are irregular.

2. *Read the text below.*

In 1952, a major computing company made a decision to get out of the business of making mainframe computers. They believed that there was only a market for four mainframes in the whole world. That company was IBM. The following years they reversed their decision.

In 1980, IBM determined that there was a market for 250,000 PCs, so they set up a special team to develop the first IBM PC. It went on sale in 1987 and set a world wide standard for compatibility i.e. IBM-compatible as opposed the single company Apple computers standard. Since then, over seventy million IBM-compatible PCs, made by IBM and other manufacturers, have been sold.

Work in pairs

Ask and answer questions about the text.

Example: What did IBM company decide to do in 1952?

– They decided to get out of the business of making mainframe computers.

• **Grammar questions**

– Why is the past simple tense used in the text?

– How do we form questions?

– How do we form negatives?

PRACTICE

1. Grammar

The present simple or the past simple. Put the verbs in brackets in the correct forms.

- a. The problem of constructing a regular polygon of nine sides which(require) the trisection of a 60⁰ angle (be) the second source of the famous problem.
- b. The Greeks (add) “the trisection problem” to their three famous unsolved problems. It (be) customary to emphasize the futile search of the Greeks for the solution.
- c. The widespread availability of computers (have) in all, probability changed the world for ever.
- d. The microchip technology which (make) the PC possible has put chips not only into computers, but also into washing machines and cars.
- e. Fermat almost certainly (write) the marginal note around 1630, when he first (study) Diophantus’s Arithmetica.
- f. I (protest) against the use of infinitive magnitude as something completed, which (be) never permissible in maths, one (have) in mind limits which certain ratio (approach) as closely as desirable while other ratios may increase indefinitely (Gauss).
- g. In 1676 Robert Hooke(announce) his discovery concerning springs. He(discover) that when a spring is stretched by an increasing force, the stretch varies directly according to the force.

2. Pronunciation

There are three pronunciations of

the past tense ending –ed: /t /, /id /, /d /.

Put the regular past tense form in exercise 1 into the correct columns. Give more examples.

/ t /	/ id /	/ d /
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

3. Writing

Put the sentences into the right order to make a complete paragraph.

WHAT IS MATHEMATICS ?

The largest branch is that which builds on ordinary whole numbers, fractions, and irrational numbers, or what is called collectively the real number system.

Hence, from the standpoint of structure, the concepts, axioms and theorems are the essential components of any compartment of maths.

1 Maths, as science, viewed as whole, is a collection of branches.

These concepts must verify explicitly stated axioms. Some of the axioms of the maths of numbers are the associative, commutative, and distributive properties and the axioms about equalities.

Arithmetic, algebra, the study of functions, the calculus differential equations and other various subjects which follow the calculus in logical order are all developments of the real number system. This part of maths is termed the maths of numbers.

Some of the axioms of geometry are that two points determine a line, all right angles are equal, etc. From these concepts and axioms, theorems are deduced.

A second branch is geometry consisting of several geometries. Maths contains many more divisions. Each branch has the same logical structure: it begins with certain concepts, such as the whole numbers or integers in the maths of numbers or such as points, lines, triangles in geometry.

- **Speaking and listening**

Work in pairs to ask and answer the question about the text in exercise 3.

For example: How many branches are there in maths?
 What are they?

Speaking

a. Learn how to say these following in English.

- | | | | | |
|----------------------|------------------|--------------|--------------|-----------|
| 1) \equiv | 4) \rightarrow | 7) \square | 10) \geq | 13) \pm |
| 2) \neq | 5) $<$ | 8) \square | 11) α | 14) $/$ |
| 3) $\square \approx$ | 6) $>$ | 9) \leq | 12) ∞ | |

b. Practice saying the Greek alphabet.

α A	η H	ν N	τ T
β B	θ Θ	ξ Ξ	υ Υ
γ Γ	ι I	\omicron O	φ Φ
δ Δ	κ K	π Π	ζ Ζ
ε E	λ Λ	ρ Ρ	ψ Ψ
ς Ζ	μ Μ	σ Σ	ω Ω

SKILLS DEVELOPMENT

• **Reading**

1. Pre – reading task

1.1 Use your dictionary to check the meaning of the words below.

triple (adj.)	utilize (v.)
conjecture (v.)	bequeath (v.)
conjecture (n.)	tarnish (v.)
subsequent (adj.)	repute (v.) [be reputed]

1.2 Complete sentences using the words above.

- The bus is traveling at..... the speed.
- What the real cause was is open to.....
-events proved me wrong.
- He is..... as / to be the best surgeon in Paris.
- People've solar power as a source of energy.
- Discoveries..... to us by scientists of the last century.
- The firm's good name was badly.....by the scandal.

2. Read the text.

FERMAT'S LAST THEOREM

Pierre de Fermat was born in Toulouse in 1601 and died in 1665. Today we think of Fermat as a number theorist, in fact as perhaps the most famous number theorist who ever lived.

The history of Pythagorean triples goes back to 1600 B.C, but it was not until the seventeenth century A.D that mathematicians seriously attacked, in general terms, the problem of finding positive integer solutions to the equation $x^n + y^n = z^n$. Many mathematicians conjectured that there are no positive integer solutions to this equation if n is greater than 2. Fermat's now famous conjecture was inscribed in the margin of his copy of the Latin translation of



Diophantus's *Arithmetica*. The note read: *"To divide a cube into two cubes, a fourth power or in general any power whatever into two powers of the same denomination above the second is impossible and I have assuredly found an admirable proof of this, but the margin is too narrow to contain it"*.

Despite Fermat's confident proclamation the conjecture, referred to as "Fermat's last theorem" remains unproven. Fermat gave elsewhere a proof for the case $n = 4$. It was not until the next century that L.Euler supplied a proof for the case $n = 3$, and still another century passed before A.Legendre and L.Dirichlet arrived at independent proofs of the case $n = 5$. Not long after, in 1838, G.Lame established the theorem for $n = 7$. In 1843, the German mathematician E.Kummer submitted a proof of Fermat's theorem to Dirichlet. Dirichlet found an error in the argument and Kummer returned to the problem. After developing the algebraic "theory of ideals", Kummer produced a proof for "most small n ". Subsequent progress in the problem utilized Kummer's ideals and many more special cases were proved. It is now known that Fermat's conjecture is true for all $n < 4.003$ and many special values of n , but no general proof has been found.

Fermat's conjecture generated such interest among mathematicians that in 1908 the German mathematician P.Wolfskehl bequeathed DM 100.000 to the Academy of Science at Gottingen as a prize for the first complete proof of the theorem. This prize induced thousands of amateurs to prepare solutions, with the result that Fermat's theorem is reputed to be the maths problem for which the greatest number of incorrect proofs was published. However, these faulty arguments did not tarnish the reputation of the genius who first proposed the proposition – P.Fermat.

Comprehension check

1. Answer the following questions.

- How old was Pierre Fermat when he died?
- Which problem did mathematicians face in the 17 century A.D?
- What did many mathematicians conjecture at that time?
- Who first gave a proof to Fermat's theorem?
- What proof did he give?
- Did any mathematicians prove Fermat's theorem after him? Who were they?

2. Are the statements True (T) or False (F)? Correct the false sentences.

- The German mathematician E.Kummer was the first to find an error in the argument.
- With the algebraic "theory of ideals" in hand, Kummer produced a proof for "most small n " and many special cases.
- A general proof has been found for all value of n .
- The German mathematician P.Wolfskehl won DM 100.000 in 1908 for the first complete proof of the theorem.

3. Discussion

Discuss in groups the following question.

What is the famous Fermat's theorem you've known?

- **Listening and writing**

You will hear some equations in words. Write them in formulae.

- a. F equals one over two pi times the square root of LC.....
- b. E equals sigma T to the power of four.....
- c. Capital W subscript *s* equals two pi small *f* over capital P.....
- d. Gamma equals W subscript oh over four pi R all times F.....
- e. Mu subscript oh equals four pi times ten to the power of minus seven capital H small *m* to the power of minus one.....
- f. C equals L over R squared plus omega squared L squared.....
- g. V subscript two equals the squared root of open brackets, two *e* over *m* times capital V subscript two, close brackets.....
- h. Sigma equals capital M small *y* small *c* over capital I, plus capital P over capital A.....
- i. Gamma equals four Q over three pi R squared times, open brackets, R squared minus gamma squared, close brackets.....

TRANSLATION

- *Translate into Vietnamese.*

1. Fermat's theorem

The theorem that if a is an integer and p is a prime that does not divide a , then p does divide $a^{p-1}-1$; or in congruence notation, $a^{p-1} \equiv 1 \pmod{p}$. For example, $8^4 - 1$ is divisible by 5. A simple corollary is that, whether p divides a or not, it must divide $a^p - a$: equivalently $a^p \equiv a \pmod{p}$.

2. Fermat's last theorem

The conjecture that if the integer n is at least 3 then there are no integers x, y, z , none of which is zero, satisfying: $x^n + y^n = z^n$

Work on Fermat's last theorem has provided much stimulus to the development of algebraic number theory; the impossibility of finding non zero integers x, y, z to satisfy the given equation has now been established for every n between 3 and 125000 inclusive.

- *Translate into English and explain what happened.*

Bãin ñãõ queãn vaøi thöù. Ñãây laø söi khai trieãn thuù vò trong ñãõi soá sô caáp.

Cho	:	$a = b$
Nhaân 2 veá vôi a	:	$a^2 = ab$
Tröø 2 veá vôi b^2	:	$a^2 - b^2 = ab - b^2$
Phaân tích thaønh thöøa soá	:	$(a - b)(a + b) = b(a - b)$
Chia 2 veá cho $a - b$:	$a + b = b$
Thay theá $a = b$:	$b + b = b \quad \rightarrow \quad 2b = b$
Chia 2 veá cho b	:	$2 = 1$

Chuyeãn gì ñãõ xaây ra?

Just for fun

☺ THE USE OF FOREIGN LANGUAGE

- Little mouse: – Mommy! He’s saying something that I don’t understand at all?
 Mother mouse: – Silence! It’s our enemy. Don’t go out of the house. That dirty cat is threatening us.
 Little mouse: – How did you understand what he said?
 Mother mouse: – Consider it a very good reason to learn a foreign language.

UNIT 3

THE PRESENT PERFECT

PRESENTATION

1. Read the text about “Fractions”.

FRACTIONS

Every fraction has a numerator and denominator. The denominator tells you the number of parts of equal size into which some quantity is divided. The numerator tells you how many of these parts are to be taken.

Fractions representing values less than 1, like $\frac{2}{3}$ for example, are called proper fractions. Fractions which name a number equal to or greater than 1, like $\frac{2}{2}$ or $\frac{3}{2}$, are called improper fractions. There are numerals like $1\frac{1}{2}$, which name a whole number and a fractional number. Such numerals are called mixed fractions. Fractions which represent the same fractional number like $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$ and so on, are called equivalent fractions.

We have already seen that if we multiply a whole number by 1 we leave the number unchanged. The same is true of fractions when we multiply both integers in a fraction by the same number. For example, $1 \times \frac{1}{2} = \frac{1}{2}$. We can also use the idea that 1 can be expressed as a fraction in various ways $\frac{2}{2}, \frac{3}{3}, \frac{4}{4}$ and so on.

Now see what happens when you multiply $\frac{1}{2}$ by $\frac{2}{2}$. You will have

$$\frac{1}{2} = 1 \times \frac{1}{2} = \frac{2}{2} \times \frac{1}{2} = \frac{2 \times 1}{2 \times 2} = \frac{2}{4}.$$

As a matter of fact in the above operation you have changed the fraction to its higher terms.

Now look at this: $\frac{6}{8} : 1 = \frac{6}{8} : \frac{2}{2} = \frac{6:2}{8:2} = \frac{3}{4}$. In both of the above operations the number you have chosen for 1 is $\frac{2}{2}$.

In the second example you have used division to change $\frac{6}{8}$ to lower terms, that is to $\frac{3}{4}$. The numerator and denominator in this fraction are prime and accordingly we call such a fraction the simplest fraction for the given rational number.

2. Grammar questions

2.1 Why is the Present Perfect tense used in the text?

2.2 Complete the rule:

The Present Perfect is formed with the auxiliary verb..... + the.....

• Speaking

Work in pairs

Answer the questions below.

- a. What have you seen if you multiply a whole number by 1?
- b. Have you changed the fraction when you multiply $\frac{1}{2}$ by $\frac{2}{2}$?
- c. What division have you used to change $\frac{6}{8}$ to lower terms?

PRACTICE

1. Grammar

1.1 Read the following newspaper extracts and say when these actions took place. If you do not have information, write "don't know".

(Financial Times, page 5, 31 January 1994)

China (1) **has extended** its freeze on new capital spending projects until the end of this year as part of an effort to combat inflation. The State Council, China's cabinet, (2) **announced** at the weekend that 'no new fixed-asset investment projects' would be approved.

(The Times, page 21, 29 January 1994)

The seemingly unstoppable success story of J Sainsbury, Britain's biggest supermarket group, (3) **came** to an abrupt halt yesterday when they (4) **warned** that profits in the current year would be substantially lower than market expectations. The news (5) **hit** Sainsbury's shares, which (6) **plummeted** from 481p to 393p.

(Financial Times, page 14, 31 January 1994)

Two years ago it (7) **seemed** as though Mr Trump might no longer have his desk, his office, his tower, or any of the rest of the property and casino empire he (8) **built up** during the 1980s. And yet, he (9) **has survived**. Helped by the cashflows from his casinos, he (10) **has paid** off a large part of his debt.

- | | | | |
|-----------------|-----------------------|------------------|-------|
| 1. has extended | <u>don't know</u> | 6. plummeted | |
| 2. announced | <u>at the weekend</u> | 7. seemed | |
| 3. came | | 8. built | |
| 4. warned | | 9. has survived | |
| 5. hit | | 10. has paid off | |

1.2 Underline the correct tense.

- If you *write / have written* first $\frac{1}{2} = \frac{3}{6}$ and then $\frac{1}{2} = \frac{n}{6}$ that means you *replace / have replaced* 3 with *n*.
- My research adviser *find / has found* the second chapter of my dissertation too long.
- You *do not divide / have not divided* the given quantity into two parts.
- The professors *agree / have agreed* to accept these principles as the basis of their work.
- Some first year students *perform / have performed* this relativity simple operation.

◆ **Note**

The Present Perfect is also used to express complete actions over a period of time.

1.3 Put the verbs in the brackets into the Present Perfect tense and read through this extract from an advertisement about the Emerging Markets Fund.

Over the past five years, the capital returns from many emerging Asian and Latin American stock markets (be) substantially higher than those of the developed world.

For example the market in Argentina(rise) by 793% and Mexico (increase) by 645%. In Asia, the booming market in Thailand (go up) by 364%,and investors in the Philippines (see) a return of 204%. The major developed nations (not / manage) to make anything like such significant returns. The market in the USA (grow) by 69.8% and in Japan, the market (fall) by 32.2% over the same period. The growth rates that these emerging markets (enjoy) in recent years is little short of phenomenal. And we are firmly convinced, much more is yet to come. Our new Emerging Markets Fund, therefore, offers you an easy and attractive way of investing now in the world of tomorrow and its many exceptional growth opportunities.

2. Speaking

Practice saying these expressions.

2.1 *Fractions*

$$\frac{1}{2} , \frac{1}{4} , \frac{1}{3} , \frac{2}{3} , \frac{3}{4} , \frac{1}{8} , \frac{3}{16}$$

2.2 Equations

$$a) x = \frac{a+b}{c}$$

$$d) V = IR$$

$$b) x + y = \frac{\Delta}{a-b}$$

$$e) \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$c) I = a + (n-1)d$$

$$f) v = u + at$$

SKILLS DEVELOPMENT

• Reading and speaking

1. Pre – reading task

Discuss in groups the following problem.

What is the difference (distinction) between two math terms: “natural numbers” and “cardinal numbers”. Is the number 5 natural or cardinal?

2. Reading

Read the text to get more information about J.E.Freund’s System of Natural Numbers Postulates.

J.E.FREUND’S SYSTEM OF NATURAL NUMBERS POSTULATES

Modern mathematicians are accustomed to derive properties of natural numbers from a set of axioms or postulates (i.e., undefined and unproven statements that disclose the meaning of the abstract concepts).

The well known system of 5 axioms of the Italian mathematician, Peano provides the description of natural numbers. These axioms are:

First: 1 is a natural number.

Second: Any number which is a successor (follower) of a natural number is itself a natural number.

Third: No two natural numbers have the same follower.

Fourth: The natural number 1 is not the follower of any other natural number.

Fifth: If a series of natural numbers includes both the number 1 and the follower of every natural number, then the series contains all natural numbers.

The fifth axiom is the principle (law) of math induction.

From the axioms it follows that there must be infinitely many natural numbers since the series cannot stop. It cannot circle back to its starting point either because 1 is not the immediate follower of any natural number. In essence, Peano’s theory states that the series of natural numbers is well ordered and presents a general problem of quantification. It places the natural numbers in an ordinal relation and the commonest example of ordination is the counting of things. The domain of applications of Peano’s theory is much wider than the series of natural numbers alone e.g., the relational fractions $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and so on, satisfy the axioms similarly. From Peano’s five rules

we can state and enumerate all the familiar characteristics and properties of natural numbers. Other mathematicians define these properties in terms of 8 or even 12 axioms (J.E.Freund) and these systems characterize properties of natural numbers much more comprehensively and they specify the notion of operations both arithmetical and logical.

Note that sums and products of natural numbers are written as $a + b$ and $a \cdot b$ or ab , respectively.

Postulate No.1: For every pair of natural numbers, a and b , in that order, there is a unique (one and only one) natural number called the sum of a and b .

Postulate No.2: If a and b are natural numbers, then $a + b = b + a$

Postulate No.3: If a , b and c are natural numbers, then

$$(a + b) + c = a + (b + c)$$

Postulate No.4: For every pair of natural numbers, a and b , in that order, there is a unique (one and only one) natural number called the product.

Postulate No.5: If a and b are natural numbers, then $ab = ba$

Postulate No.6: If a , b and c are natural numbers, then $(ab)c = a(bc)$

Postulate No.7: If a , b and c are natural numbers, then $a(b + c) = ab + ac$

Postulate No.8: There is a natural number called “one” and written 1 so that if a is an arbitrary natural number, then $a \cdot 1 = a$

Postulate No.9: If a , b and c are natural numbers and if $ac = bc$ then $a = b$

Postulate No.10: If a , b and c are natural numbers and if $a + c = b + c$ then $a = b$

Postulate No.11: Any set of natural numbers which (1) includes the number 1 and which (2) includes $a + 1$ whenever it includes the natural number a , includes every natural number.

Postulate No.12: For any pair of natural numbers, a and b , one and only one of the following alternatives must hold: either $a = b$, or there is a natural number x such that $a + x = b$, or there is a natural number y such that $b + y = a$.

Freund’s system of 12 postulates provides the possibility to characterize natural numbers when we explain how they behave and what math rules they must obey. To conclude the definition of “natural numbers” we can say that they must be interpreted either as standing for the whole number or else for math objects which share all their math properties. Using these postulates mathematicians are able to prove all other rules about natural numbers with which people have long been familiar.

Comprehension check

1. Answer the questions.

- How many axioms did the Italian mathematician Peano give? What were they?
- Which axiom is the most important? Why?
- What does Peano’s theory state in essence?
- What can we state from Peano’s five rules?
- Who developed these axioms? What did he do?
- How useful is Freund’s system of 12 postulates?

2. Work in pairs

a. Complete the formulae written by Freund's system of 12 postulates.

If a, b, c are natural numbers:

$a + b$	=	$a(b + c)$	=
$(a + b) + c$	=	$a.1$	=
ab	=	$ac = bc$	\Rightarrow
$(ab)c$	=	$a + c = b + c$	\Rightarrow

b. Practice speaking them based on the 12 postulates.

• Listening and speaking

1. Pre – listening

The following words are used in the listening paragraph.

- | | |
|---------------------|-------------------|
| branch (n.) | set (n.) |
| collection (n.) | signify (v.) |
| capital letter (n.) | symbol (n.) |
| belong (v.) | synonymous (adj.) |

Work in pairs

Look at these symbols.

$$a \in S$$

$$b \notin S$$

What do they mean?

2. Try to fill in the gaps with the words you hear.

In all (1)..... of mathematics, we are concerned with collections of objects of one kind or another. In basic algebra (2)were the principle objects of investigation. The terms (3)..... and (4)..... are undefined but are taken to be (5).....

A description, or a property of the objects which (6)..... to a set must be clearly stated.

We note that an object which belongs to a set may itself be a set. If an object belongs to a set, it is called a (7)..... or (8)..... of that set.

The symbol $a \in S$ means that a is an element of the set S . It is customary, in elementary set theory, to denote sets by (9)..... and elements of sets by (10)..... Also $b \notin S$ signifies that b is not an element of S .

3. Give some examples of sets.

TRANSLATION

• Translate into Vietnamese.

1. A set A of real numbers is said to be inductive if, and only if, $1 \in A$ and $x \in A$ implies $(x+1) \in A$.
2. The real number system must have any property which is possessed by a field, an ordered field, or a complete ordered field.
3. A real number is called a rational number if, and only if, it is the quotient of two integers. A real number which is not rational is said to be irrational.

• *Translate into English.*

1. Nếu A và B là các tập hợp và $x \in A$ kéo theo $x \in B$ thì ta nói A là tập hợp con của B và ta viết $A \subset B$.
2. Nếu A là tập hợp con của B và có ít nhất một phần tử của B không phải là phần tử của A thì ta nói A là tập hợp con thực sự của B .
3. Hai tập hợp A và B là bằng nhau, khi và chỉ khi $A \subset B$ và $B \subset A$.

Just for fun

☺ A LESSON IN SUMMATION

- The teacher: – If your father can do a piece of work in one hour and your mother can do it in one hour, how long would it take both of them to do it?
- A pupil: – Three hours, teacher!
- The teacher: – Why?
- The pupil: – I had to count the time they would waste in arguing.

UNIT 4

DEGREES OF COMPARISON

PRESENTATION

1. Read the following text.

SOMETHING ABOUT MATHEMATICAL SENTENCES

A mathematical sentence containing an equal sign is an equation. The two parts of an equation are called its members. A mathematical sentence that is either true or false but not both is called a closed sentence. To decide whether a closed sentence containing an equal sign is true or false, we check to see that both elements, or members of the sentence name the *same* number. To decide whether a closed sentence containing an \neq sign is true or false, we check to see that both elements do not name the *same* number.

The relation of equality between two numbers satisfies the following basic axioms for the numbers a , b and c .

Reflexive: $a = a$.

Symmetric: If $a = b$ then $b = a$.

Transitive: If $a = b$ and $b = c$ then $a = c$.

While the symbol $=$ in an arithmetic sentence means *is equal to*, another symbol \neq , means *is not equal to*. When an $=$ sign is replaced by \neq sign, the opposite meaning is implied. (Thus $8 = 11 - 3$ is read eight is equal to eleven minus three while $9 + 6 \neq 13$ is read nine plus six *is not equal to thirteen*.)

The important feature about a sentence involving numerals is that it is either true or false, *but not both*. There is nothing incorrect about writing a false sentence, in fact in some mathematical proofs it is essential that you write a false sentence.

We already know that if we draw one short line across the symbol $=$ we change it to \neq . The symbol \neq implies either of two things – *is greater than* or *is less than*. In other words the sign \neq in $3 + 4 \neq 6$ tells us only that numerals $3 + 4$ and 6 name different numbers, but does not tell us which numeral names *the greater* or the *lesser* of the two numbers.

To know which of the two numbers is greater let us use the conventional symbol $<$ and $>$. $<$ means *is less than* while $>$ means *is greater than*. These are inequality symbols because they indicate order of numbers. ($6 < 7$ is read six is less than seven, $29 > 3$ is read twenty nine is greater than three). The signs which express equality or inequality ($=$, \neq , $<$, $>$) are called relation symbols because they indicate how two expressions are related.

2. Work in pairs

2.1 Express the symbol $=$, \neq , $>$, $<$ in arithmetical sentences.

Example: $x > y$: – Is x equal to y ?

– No, x is greater than y .

- a) $a = b^2$
- b) $\alpha \neq \beta$
- c) $3b + 2c > 1$
- d) $x^2 - x < 0$

2.2 How are the symbols $=$, \neq , $<$, $>$ read?

3. Grammar questions

- When do you use $-er / -est$
 $-ier / -iest$?
 $more / most$
- When do you use $as \dots as$
 $as \text{ many } \dots as$
 $as \text{ much } \dots as$
 $the \text{ same } \dots as$?
 $similar \text{ to}$
 $the \text{ same}$
- When do you use $not \text{ as } \dots as$
 $\dots-er \text{ than}$
 $more \dots than$?
 $fewer \dots than$

PRACTICE

1. Grammar

1.1 Write the comparative and superlative of the words below.

new	tiny	common	bad
soon	shallow	gentle	little
convenient	clever	badly	many
easily	complex	good	much

1.2 Write the words in brackets in the correct form of the degrees of comparison.

- a. We all use this method of research because it is (interesting) the one we followed.
- b. I could solve quicker than he because the equation given to me was.....(easy) the one he was given.
- c. The remainder in this operation of division is (great) than 1.
- d. The name of Leibnitz is (familiar) to us as that of Newton.

- e. Laptops are (powerful) microcomputers. We can choose either of them.
- f. A mainframe is..... (large) and.....(expensive) a microcomputer.
- g. One of the (important) reasons why computers are used so widely today is that almost every big problem can be solved by solving a number of little problems.
- h. Even the(sophisticated) computer, no matter how good it is, must be told what to do.

1.3 Look at the table of word processing packages below and write ten sentences comparing the products advertised.

Examples: Upword is more expensive than Just Write.

Ami Pro 2.0 has the largest spell check dictionary.

Word processors

Product	Price	Spell check dictionary size	Features										Comments	Supplier and tel. no.			
			Thesaurus	Grammar check	Mail merge	Outliner	Multiple rulers	Style sheets	Auto numbering	Multiple columns	Word count						
Ami Pro 2.0	£445	135,000	•	•	•	•	•	•	•	•	•	•	•	•	•	Drawing, charting, image processing	Lotus 0784 455445
JustWrite	£199	100,000	•	•	•	•	•	•	•	•	•	•	•	•	•	Table editor, DDE support	Symantec 0628 776343
Professional Write Plus	£249	130,000	•	•	•	•	•	•	•	•	•	•	•	•	•	Harvard graphics import	Software Publishing 0344 867100
Upword	£395	OCED	•	•	•	•	•	•	•	•	•	•	•	•	•	DDE links with DOS and Windows	Wang 081 568 9200
Word for Windows 2	£445	130,000	•	•	•	•	•	•	•	•	•	•	•	•	•	Many DTP capabilities, plus drawing and charting tools	Microsoft 0734 270000
Wordperfect for Windows	£399		•	•	•	•	•	•	•	•	•	•	•	•	•	Powerful macro command language	Wordperfect Corp. 0932 850500
Wordstar for Windows	£399		•	•	•	•	•	•	•	•	•	•	•	•	•	Keystroke compatible with Wordstar 6	Wordstar International 081 643 8866

2. Speaking

Read out these expressions.

- | | |
|-------------------|------------------------|
| $x \equiv y$ | $y \square 5$ |
| $x \neq y$ | $x \leq 10$ |
| $x \approx 10$ | $y \geq 10$ |
| $x \rightarrow 0$ | $x \alpha y$ |
| $x < 5$ | $x \rightarrow \infty$ |
| $x > 5$ | $x = \pm 2$ |
| $y \square 5$ | $x = 0$ |

3. Writing

Put the words in brackets into the correct form to make an accurate description of sizes of computers.

There are different types of computer. The (large)¹ and (powerful)² are mainframe computers. Minicomputers are (small)³ than mainframes but are still very powerful. Microcomputers are small enough to sit on a desk. They are the (common)⁴ type of computer. They are usually (powerful)⁵ than minicomputers.

Portable computers are (small)⁶ than desktops. The (large)⁷ portable is a laptop. (Small)⁸ portables, about the size of a piece of writing paper, are called notebook computers. Subnotebooks are (small)⁹ than notebooks. You can hold the (small)¹⁰ computers in one hand. They are called handheld computers or palmtop computers.

SKILLS DEVELOPMENT

• Reading and speaking

1. Pre – reading task

Sometimes you see problems expressed like this:

$$7x - 3 > 2x + 7$$

Solution: Using the properties we write in turn:

$$7x - 3 > 2x + 7$$

$$7x - 2x > 7 + 3$$

$$5x > 10$$

$$x > 2$$

The solution, then, is all values of x greater than 2.

- ❖ What is $7x - 3 > 2x + 7$ called in math?
- ❖ Which properties do you use to solve the problem?

To find more information, read the text about “Inequalities”.

2. Read the text below.

INEQUALITIES

An inequality is simply a statement that one expression *is greater than or less than another*. We have seen the symbol $a > b$, which reads “ a is greater than b ” and $a < b$, which reads “ a is less than b ”. There are many ways in which to make these statements. For example, there are three ways of expressing the statement “ a is greater than b ”:

- $a > b$ or $b < a$
- $a - b > 0$; $a - b$ is a positive number.
- $a - b = n$; n is a positive number.

If an expression is *either greater than or equal to*, we use the symbol \geq , and similarly, \leq states *is less than or equal to*. Two inequalities are *alike in sense*, or of the *same sense*, if their symbols for inequality point in the same direction. Similarly, they are *unlike*, or *opposite in sense*, if the symbols point in opposite directions.

In discussing inequalities of algebraic expressions we see that we can have two classes of them:

1. If the sense of inequality is the same for **all** values of the symbols for which its members are defined, the inequality is called an **absolute** or **unconditional inequality**.

Illustrations: $x^2 + y^2 > 0$, $x \neq 0$ or $y \neq 0$
 $\pi < 4$

2. If the sense of inequality holds only for **certain** values of the symbols involved, the inequality is called a **conditional inequality**.

Illustrations: $x + 3 < 7$, true only for values of x less than 4;
 $x^2 + 6 < 5x$, true only for x between 2 and 3.

The inequality symbols are frequently used to denote the values of a variable between given limits. Thus, $1 \leq x < 4$, states “values of x from 1, including 1, to 4 but not including 4”, i.e., x may assume the value 1 and from 1 to 4 but no others. This is also called “defining the **range of values**”.

$$x^2 + 6 < 5x \text{ for } 2 < x < 3$$

Properties:

- a. The sense of an inequality is not changed if both members are increased or decreased by the same number.

If $a > b$, then $a + x > b + x$ and $a - x > b - x$

- b. If $a > b$ and $x > 0$, then: $ax > bx$ and $\frac{a}{x} > \frac{b}{x}$

- c. If $a > b$ and $x < 0$, then: $ax < bx$ and $\frac{a}{x} < \frac{b}{x}$

- d. If a , b and n are positive numbers and $a > b$, then: $a^n > b^n$ and $\sqrt[n]{a} > \sqrt[n]{b}$

- e. If $x > 0$, $a > b$ and a, b have like signs, then: $\frac{x}{a} < \frac{x}{b}$

We can illustrate these properties by using numbers.

Illustrations:

- (1) Since $4 > 3$, we have $4 + 2 > 3 + 2$ as $6 > 5$
- (2) Since $4 > 3$, we have $4(2) > 3(2)$ as $8 > 6$
- (3) Since $4 > 3$, we have $4(-2) < 3(-2)$ as $-8 < -6$
- (4) Since $16 > 9$, we have $\sqrt{16} > \sqrt{9}$ as $4 > 3$
- (5) Since $4 > 3$, we have $\frac{2}{4} < \frac{2}{3}$ as $\frac{1}{2} < \frac{2}{3}$

The solutions of inequalities are obtained in a manner very similar to that of obtaining solutions to equations. The main difference is that we are now finding a *range* of values of the unknown such that the inequality is satisfied. Furthermore, we must pay strict attention to the properties so that in performing operations we do not change the sense of inequality without knowing it.

Comprehension check

1. Answer the questions.

- What is an inequality in maths?
- What does the following mean: $a > b$?
- Which symbol do we use to signify an expression “is either greater than or equal to”?
- When are two inequalities like or unlike in sense?
- Do you know any kinds of inequality?

2. Writing

State the expression “ a is greater than b ” in different ways.

3. Work in pairs

Illustrate 5 properties of inequality by using number.

- Since $4 > 3$, we have $4 + 2 > \dots\dots\dots$
- Since $4 > 3$, we have $\dots\dots\dots$
- Since $4 > 3$, we have $\dots\dots\dots$
- Since $16 > 9$, we have $\dots\dots\dots$
- Since $4 > 3$, we have $\dots\dots\dots$

• Listening

1. Pre – listening task

1.1 Look up in your dictionary to find the meaning of the following words.

- | | |
|---------------------|--------------|
| – remarkable (adj.) | – tend (v.) |
| – identity (n.) | – limit (n.) |

1.2 Put the correct forms of the given words into the spaces below.

- He $\dots\dots\dots$ to extreme views.
- He is a boy who is $\dots\dots\dots$ for his stupidity.
- The cheque will be cashed on proof of $\dots\dots\dots$
- No fishing is allowed within a twenty mile $\dots\dots\dots$

2. Listening

You will hear a conversation about symbols and signs in maths language. **Listen and fill in the gaps.**

Mathematical Signs and Symbols

A: How $\dots\dots\dots$ in the language of maths?

B: As far as I know, $\dots\dots\dots$, not less, and of various categories: symbols of math objects, relations and operations.

A: Which symbol or sign is $\dots\dots\dots$?

B: Certainly, it's the, which is translatable as "is another name for". Such basic maths concepts as an (e.g., $aaa = a^3$), an (e.g., $ab = ba$), an (e.g., $2x + 5 = 11$) all involve this sign.

A: The equality sign is basic in maths, sure enough. However, among numerous symbols and signs, one is....., with the big meaning: " ∞ ". How should we..... it?

B: It is "infinite". The maths notation " $\rightarrow \infty$ " must be worded

A: Is there any need to go out of this world to locate

B: In the scientist's mind, it's an abstract concept. In the calculus " ∞ " means and nothing more. In algebra we must use the symbol for a variable

3. Listen again and check your answer.

4. Post-listening

- a. Give some signs you often use in maths? Which sign do you think is the most important?
- b. How can you pronounce the symbol " ∞ "? What does it mean?

TRANSLATION

• **Translate into Vietnamese.**

One and Zero

The ONE stood itself up tall and thin as a rod, and imagining itself a flagpole, swaggered from town to town, gathering up simple minded and empty ZEROS.

"Follow me! We'll be invincible! A few friends will make us thousands, and with a few more we'll be millions!"

The mathematician watched the procession.

"Wonderful!" he laughed. "A few buglers make a big shot."

• **Translate into English.**

1. Những số lớn hơn zero là các số dương, những số nhỏ hơn zero là các số âm.
2. Nếu a, b, x là ba số nguyên dương bất kỳ, mà $a + x = b + x$ thì suy ra $a = b$.
3. Nếu a, b, x là ba số nguyên dương bất kỳ, mà $ax = bx$ thì suy ra $a = b$.

UNIT 5

- ING ENDING FORMS

PRESENTATION

1. Read the passage below.

There is much thinking and reasoning in maths. Students master the subject matter not only by reading and learning, but also by proving theorems and solving problems. The problems therefore are an important part of teaching, because they make students discuss and reason and polish up on their own knowledge. To understand how experimental knowledge is matched with theory and new results extracted, the students need to do their own reasoning and thinking.

Some problems raise general questions which discussion of, can do much to advance your understanding of particular points of the theory. Such general questions ask for opinions as well as reasoning; they obviously do not have a single, unique or completely right answer. More than that, the answers available are sometimes misleading, demanding more reasoning and further proving. Yet, thinking your way through them and making your own choices of opinion and then discussing other choices is part of a good education in science and method of teaching.

2. Underline all – ing forms in the passage.

3. Compare these sentences.

- a. Students need to do their own reasoning and thinking.
- b. Thinking your way through them and making your own choices of opinion and then discussing other choices is part of a good education in science.
- c. The answers available are sometimes misleading, demanding more reasoning and further proving.
 - i. In which sentence is the – ing form used as a subject?
 - ii. In which sentence is the – ing form used as an object?
 - iii. In which sentence does the – ing form modify a noun?

4. Complete the rules.

- a. When the – ing form is used in the same way as a, i.e. as a or, it is a gerund.
- b. When the – ing form is used as an....., i.e. it modifies a, it is a present participle, functioning as an adjective.

PRACTICE

- Grammar

1. Read the statements and identify the function of the – ing forms as a subject, an object, a complement or a modifier.

- a. Fibre optic cable can be used for linking computers.
- b. The locating point on the y–axis will give the first point on the line.
- c. The difference can be found from performing the operation of subtraction.
- d. He succeeded in coming on time.
- e. His drawing line has cut the segment exactly.

2. Complete the sentences with the – ing form of an appropriate verb from the list.

perform – find – link – know – keep up – receive – multiply – send

- a. with the latest news of your favourite team is easy on the Web.
- b. They have circuits for arithmetic operations.
- c. One of the most useful features of the Internet is andemail.
- d. The product may be found by the factors contained in the given mathematical sentence.
- e. Search engines are a way of information on the Web.
- f. the properties of equality will help you decide whether a sentence is true or false.

3. Speaking

Work in pairs to make a conversation following the example.

Example: draw pictures on a computer? (use graphic package)

- – How do you draw pictures on a computer?
- By using a graphic package.

- a. find a website? (use a search engine)
- b. select an option on a menu? (click the mouse)
- c. increase the speed of your computer? (add memory)
- d. end a search on the web? (select the stop button)
- e. form a ray? (extend a line segment in only one direction)
- f. locate the point in the plane? (apply your knowledge of geometry)

4. Continue the conversation and practise in pairs.

Example: A: It is important to know these rules.
B: Yes, knowing these rules is important.

- a. A: It is difficult to locate the point in space.
B: Yes,
- b. A: It was very necessary to produce that information.
B: Yes,
- c. A: It will be interesting to find that result.
B: Yes,
- d. A: It is important to discuss the problem today.
B: Yes,

- e. A: It was easy to solve the problem.
 B: Yes,

• **Vocabulary**

Indefinite pronouns

Indefinite pronouns refer to people or things generally rather than specifically. They are used when the speaker or writer does not know or doesn't have to say exactly who or what is referred to.

1. *Complete the column with the indefinite pronouns.*

	one	body	thing	where
some	someone	somebody	something	somewhere
any				
no				
every				

2. *Fill in the gaps with one of the definite pronouns above.*

- We know about his work.
- Was there sign between these numerals?
- confuses these basic terms.
- is ready for the experiment.
- Did you find the same result
- knows this familiar theorem.
- There is not here who knows this subject.
- You can find this book
- There are not books on mathematics there.
- of our students, will take their exams today.

SKILLS DEVELOPMENT

• **Reading**

1. **Pre – reading task**

Here are some words related to geometry:

point, endpoint, line, line segment, ray, subset.

Check that you understand their meanings. Fill in the gaps using the words above.

- We can name the by using Latin alphabet letters.
- \leftrightarrow is the symbol of AB.
- The symbol \overline{AB} is used for AB.
- The two at each end of a segment are called
- \overline{AB} is used for AB.
- or measuring their land, for building pyramids, and for defining volumes. The Egyptians were mostly concerned with applying geometry to their everyday problems. A part of a line is a of a line.

2. Read the passage below.

POINTS AND LINES

Geometry is a very old subject. It probably began in Babylonia and Egypt. Men needed practical ways for, as the knowledge of the Egyptians spread to Greece, the Greeks found the ideas about geometry very intriguing and mysterious. The Greeks began to ask “Why? Why is that true?”. In 300 B.C all the known facts about Greek geometry were put into a logical sequence by Euclid. His book, called Elements, is one of the most famous books of mathematics. In recent years, men have improved on Euclid’s work. Today geometry includes not only the shape and size of the earth and all things on it, but also the study of relations between geometric objects. The most fundamental idea in the study of geometry is the idea of a point and a line.

The world around us contains many physical objects from which mathematics has developed geometric ideas. These objects can serve as models of the geometric figures. The edge of a ruler, or an edge of this page is a model of a line. We have agreed to use the word line to mean straight line. A geometric line is the property these models of lines have in common; it has length but no thickness and no width; it is an idea. A particle of dust in the air or a dot on a piece of paper is a model of a point. A point is an idea about an exact location; it has no dimensions. We usually use letters of the alphabet to name geometric ideas. For example, we speak of the following models of point as point A, point B and point C.

.B

A.

C

We speak to the following as line AB or line BA.

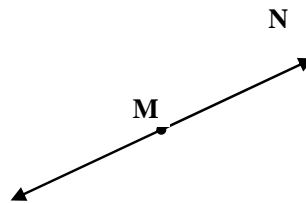


The arrows on the model above indicate that a line extends indefinitely in both directions. Let us agree to use the symbol \leftrightarrow to name a line. \overleftrightarrow{AB} means line AB. Can you locate a point C between A and B on the drawing of \overleftrightarrow{AB} above? Could you locate another point between B and C? Could you continue this process indefinitely? Why? Because between two points on a line there is another point. A line consists of a set of points. Therefore a piece of the line is a subset of a line. There are many kinds of subsets

of a line. The subset of \overline{AB} shown above is called a line segment. The symbol for the line segment AB is \overline{AB} . Points A and B are the endpoints, as you may remember. A line segment is a set of points on the line between them. How do line segment differ from a line? Could you measure the length of a line? Of a line segment? A line segment has definite length but a line extends indefinitely in

each of its directions.

Another important subset of a line is called a ray. That part of \overline{MN} shown below is called ray MN. The symbol for ray MN is \overrightarrow{MN} .



A ray has indefinite length and only one endpoint. The endpoint of a ray is called its vertex. The vertex of \overrightarrow{MN} is M. In the drawings above you see pictures of a line, a line segment and a ray – not the geometric ideas they represent.

Comprehension check

1. Are the statements True (T) or False (F)? Correct the false sentences.

- a. A point is an idea about any dot on a surface.
- b. A point does not have exact dimension and location.
- c. We can easily measure the length, the thickness and the width of a line.
- d. A line is limited by two endpoints.
- e. A line segment is also a subset of a line.
- f. Although a ray has an endpoint, we cannot define its length.

2. Answer the following questions.

- a. Where did the history of geometry begin?
- b. Who was considered the first starting geometry?
- c. What was the name of the mathematician who first assembled Greek geometry in a logical sequence?
- d. How have mathematicians developed geometric ideas?
- e. Why can you locate a point C between A and B on the line \overline{AB} ?
- f. How does a line segment differ from a line, a ray?

3. Writing

Based on the reading text, continue the following definitions.

- a. A point is
- b. A line is
- c. A line segment is

d. A ray is

● **Listening**

How to find a Website for information that you need.

1. Pre – listening task

- Have you ever found a Website for your information? Is it easy to do?
- Check the meaning of these phrases. Put them in the correct order to describe stages for finding a Website:
 - click a search button – fill in a form
 - click on the bookmark – search the Web
 - display a webpage – store a hyperlink

2. Listen to the tape. Complete the sentences and put them in the correct logical order.

- _____ **1** for the information you need by using a
- _____ One of the search engine is called
- _____ After searching the Web, it to the websites that contain the information you are looking for.
- _____ to the webpage in a or of the
- _____ on a search webpage to indicate what you're looking for. Then a search..... to start the search engine.
- _____ To return to the Webpage, on the

TRANSLATION

Translate into Vietnamese.

1. The geometry of line – called one–dimensional geometry – includes the study of lines, rays, and line segments.
2. A line is determined by any two distinct points on it. For example, \overline{AB} , \overline{BA} , \overline{CA} , \overline{AC} , \overline{BC} , \overline{CB} are different designations for the line shown in figure 1. Similarly, \overline{EF} and \overline{EG} are different designation for the ray shown in figure 2. Finally, the line segment shown in figure 3 may be designated either as \overline{MN} or \overline{NM} . It can be seen that the two points M and N completely determine the line segment.

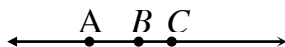


Fig.1



Fig.2

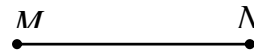


Fig.3

UNIT 6

MODAL VERBS

PRESENTATION

1. *Read the passage.*

SOME ADVICE FOR BUYING A COMPUTER

Computers can do wonders, but they can also waste a lot of your money unless careful consideration goes into buying them. People thinking of buying a computer system should admit that they know very little about computers. They must realize that the computer sales people don't always know how their business works.

It is essential that buyers should get outside advice, not necessarily from consultants but from other executives who have had recent experience in buying a computer system. Also, they have to see systems similar to ones under consideration in operation. Because their operations will have differences that must be accommodated, they should find out what would be involved in upgrading a system.

The important thing to know before buying a computer is the financial situation of the supplier because computer companies come and go and not all are financially stable. The prospective buyer should demand that every detail be covered in writing, including hardware and software if they are supplied by different companies. There's nothing wrong with computers themselves, it's how and why they are used that can cause problems.

Underline all the modal verbs in the text.

2. Grammar questions

a. *Look at the sentences below.*

- Buyers should get outside advice.
- They have to see systems similar to ones under consideration, in operation.
- They must realize that the computer sales people don't always know how their business works.

Complete the rules.

- Should, must, have to are used with
- They are forms in all persons, except

b. Work in pairs

Answer these questions.

- Which modal verb is used to give advice or mild obligation?
- Which ones are used to express strong obligation?

◆ Note 1

- a. *Must* and *have to* are used to express strong obligation but *must* is personal. We usually use *must* when we give our personal feelings:

Example: – I haven't seen Ann for ages. I must phone her tonight.

Have to is used for facts, a rule or a law.

Example: – When you drive in England, you have to drive on the left.

b. For a past obligation, it's necessary to use **had to**.

Must can not be used to mean a past obligation.

Example: – He had to phone her late at night to talk about that.

c. The negatives "**mustn't**" and "**don't have to**" also have different meanings.

+ You **mustn't** do something = It's necessary for you not to do it:

– It's a secret. You **mustn't** tell anyone.

+ You **don't have to** do something = You **don't** need to do it but you can if you want.

– I **don't have to** get up early at weekends.

PRACTICE

1. Grammar

Complete sentences using should, must or have to with the verb in brackets.

- It has been required that he(read) his paper at the seminar.
- After finding the solution, we(say) that axiom and its properties are important enough.
- Scientists(develop) this branch of mathematics, I think.
- She(summarize) the result before she reports it to her boss.
- You(distinguish) between maths objects e.g. numbers, sets of numbers, functions, mappings, transformations, etc.
- The two rays of an angle(not lie) on the same straight line.
- I think you(illustrate) this problem in the figure. This may be the easiest way.
- In geometry, set notation and language (clarify) matters.
- A polygon (not have) less than 3 segments.

[

2. Speaking

Work in groups

Use the words and phrases given below.

– do your homework regularly / read books / go to the library / prepare your lessons / search information in the Web ...

sit in rows / sit in particular seat / listen carefully to the teacher / take note / discuss / practice / ask the teacher ...

Discuss the two questions.

- What should you do to succeed in your studying?
- What do you have to do when you are in class?

[

◆ **Note 2**

The modal verbs are also used in the structure:

Modal + have + past participle

+ **Could / can / may / might + have + past participle** is used to indicate a past possibility or a possibility in the present.

Example: A pictorial representation of polygons could have been given in figures.

+ **Must + have + past participle** expresses a logical conclusion. The speaker assumes something to be true from the facts that are available but not absolutely correct.

Example: He has well informed of the theorem, so his solution must have been true.

[

PRACTICE

1. Fill in the gaps using a modal + have + past participle.

[

- a. Algebraic formulas for finding the volumes of cylinders and sphere(be used) in Ancient Egypt to compute the amount of grain contained in them.
- b. The discovery of the theorem of Pythagoras(hardly make) by Pythagoras himself, but it was certainly made in his school.
- c. Regardless of what mystical reasons(motivate) the early Pythagorean investigators, they discovered many curious and fascinating number properties.
- d. Imaginary numbers(be looking) like higher magic to many eighteenth century mathematicians.
- e. The symbol $\sqrt{\quad}$ (be used) in the sixteenth century and it resembled a manuscript form of the small *r* (radix), the use of the symbol $\sqrt{\quad}$ for square root had become quite standard.
- f. Descartes' geometric representation of negative numbers (help) mathematicians to make negative numbers more acceptable.

2. Speaking

Two colleagues are rearranging a meeting. **Complete the conversation** with: **can / can't , be able to / been able to** and then work in pairs to practice the dialogue.

Helen: Jane, I'm afraid that I won't be able to see you on Friday. I've got to see some clients and they _____ make it any other time.

Jane: Don't worry, we _____ easily meet next week. How would Tuesday morning suit you?

Helen: That's fine. I _____ come and pick you up at the station.

Jane: That's very kind of you, but my car will be back from the garage, so will _____ drive up.

Helen: I'm sorry about the delay.

Jane: That's fine, really. I haven't _____ do much work on the proposal, and now I've got an extra weekend. I'll look at it in more detail.

SKILLS DEVELOPMENT

• Reading

1. Pre – reading task

1.1 Use your dictionary to check the meaning of these words.

theorem (n.)

stretch (v.)

total (adj.)

region (n.)

ancient (adj.)

area (n.)

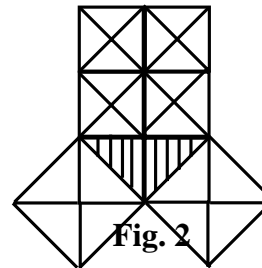
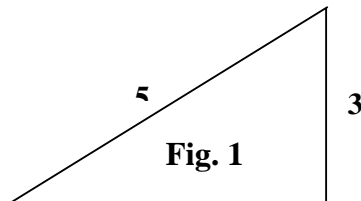
1.2 Fill in the gaps using the words above.

- Why do you have to these ropes?
– To hang up these wet clothes.
- The proof of the stated seemed rather complicated.
- The Egyptians believed that light travels from our eyes to the objects we look at, rather than from the objects to our eyes.
- The kitchen has a / an of 12 square metres.
- The sum of the four triangles makes the area of this square.
- This of the area is dashed.

2. Read the text below.

THE PYTHAGOREAN PROPERTY

The ancient Egyptians discovered that in stretching ropes of lengths 3 units, 4 units and 5 units as shown below, the angles formed by the shorter ropes is a right angle (Figure.1). The Greeks succeeded in finding other sets of three numbers which gave right triangles and were able to tell without drawing the triangles which ones should be right triangles, their method being as follow. If you look at the illustration you will see a triangle with a dashed interior (Figure.2).



Each side of the triangle is used as the side of a square. Count the number of small triangular regions in the two smaller squares then compare with the number of

triangular regions in the largest square. The Greek philosopher and mathematician Pythagoras noticed the relationship and was credited with the proof of this property. Each side of right triangle was used as a side of a square, the sum of the areas of the two smaller squares is the same as the area of the largest square.

Proof of the Pythagorean Theorem

We would like to show that the Pythagorean Property is true for all right angle triangles, there are several proofs of this property.

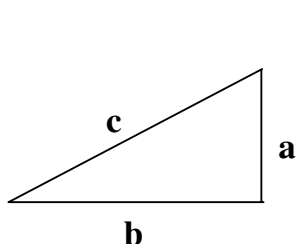


Fig. 3

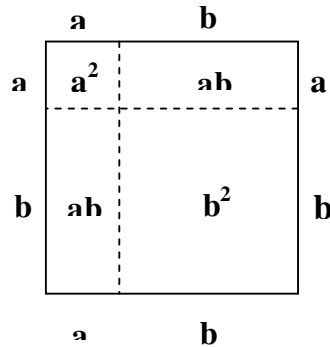


Fig. 4

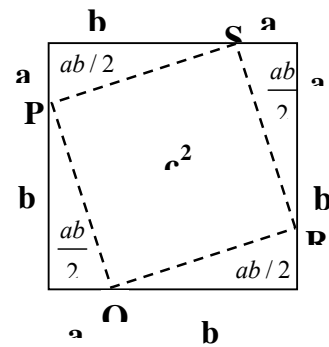


Fig. 5

Let us discuss one of them. Before giving the proof let us state the Pythagorean Property in mathematical language. In the triangle (Figure.3), c represents the measure of the hypotenuse, and a and b represent the measures of the other two sides. If we construct squares on the three sides of the triangle, the area – measure will be a^2 , b^2 and c^2 . Then the Pythagorean Property could be stated as follows: $c^2 = a^2 + b^2$. This proof will involve working with areas. To prove that $c^2 = a^2 + b^2$ for the triangle above, construct two squares each side of which has a measure $a + b$ as shown in figure 4 and figure 5.

Separate the first of the two squares into two squares and two rectangles as shown in figure 4. Its total area is the sum of the areas of the two squares and two rectangles.

$$A = a^2 + 2ab + b^2$$

In the second of two squares construct four right triangles as shown in figure 5. Are they congruent? Each of the four triangles being congruent to the original triangle, the hypotenuse has a measure c . It can be shown that PQRS is a square, and its area is c^2 . The total area of the second square is the sum of the areas of the four triangles and the square PQRS.

$$A = c^2 + 4\left(\frac{1}{2}ab\right)$$

The two squares being congruent to begin with, their area measures are the same. Hence we may conclude the following:

$$a^2 + 2ab + b^2 = c^2 + 4\left(\frac{1}{2}ab\right)$$

$$(a^2 + b^2) + 2ab = c^2 + 2ab$$

By subtracting $2ab$ from both area measures we obtain $a^2 + b^2 = c^2$ which proves the Pythagorean Property for all right triangles.

Comprehension check

1. Which sentences in the text answer these questions.

- Could the ancient Greeks tell the actual triangles without drawing?
Which ones would be right triangles?
- Who noticed the relationship between the number of small triangular regions in the two smaller squares and in the largest square?
- Is the Pythagorean Property true for all right triangles?
- What must one do to prove that $c^2 = a^2 + b^2$ for the triangle under consideration?
- What is the measure of the hypotenuse in which each of the four triangles is congruent to the original triangle?

2. Choose the main idea of the text.

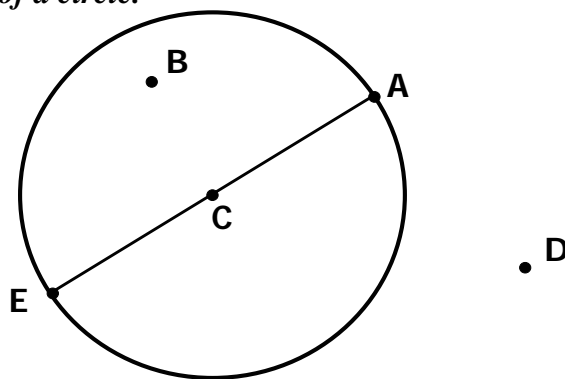
- The Pythagorean theorem is true for all right triangles and it could be stated as follows: $c^2 = a^2 + b^2$.
- The text shows that the Pythagorean Property is true for all right triangles.
- The Greek mathematician, Pythagoras contributed to maths history his famous theorem which was proved to be true for all right triangles.

3. Speaking

Work in pairs. Prove the Pythagorean Theorem using numbers.

• Listening and speaking

1. Look at the figure of a circle.



2. Listen to the tape and complete the description of this circle.

- A circle is a all of which are at a fixed distance from a
- The given point C is
- The line segment \overline{AC} is
- We call the line segment \overline{EA}
- The line that goes around the circle is called
- Point A and point E are said to be the circle.

- g. Point B and point C are of the circle.
- h. Point D is to the circle.

3. Speaking

Try to describe a circle using a figure of it.

TRANSLATION

Translate into Vietnamese.

1. An angle is the union of two rays which have a common endpoint but which do not lie on the same line.
2. Since an angle is a union of two sets of points, it is itself a set of points. When we say “the angle ABC” we are talking about a set of points – the points lying on the two rays.
3. Two angles occur so often in geometry that they are given special names. An angle of 90° is called a right angle and an angle of 180° is called a straight angle.

Just for fun ☺

Teacher: If the size of an angle is 90° , we call it a right angle.

Pupil: Then teacher, should we call all other angles wrong angles?

UNIT 7

INFINITIVE after ADJECTIVES – INFINITIVE of PURPOSE

PRESENTATION 1

Read the following passage.

MATHEMATICAL LOGIC

In order **to communicate** effectively, we must agree on the precise meaning of the terms which we use. **It's necessary to define** all terms to be used. However, **it is impossible to do this** since **to define** a word we must use others words and thus circularity can not be avoided. In mathematics, we choose certain terms as undefined and define the others by using these terms. Similarly, as **we are unable to define** all terms, we can not prove the truth of all statements. Thus we must begin by assuming the truth of some statements without proof. Such statements which are assumed to be true without proof are called *axioms*. Sentences which are proved to be laws are called *theorems*. The work of a mathematician consists of proving that certain sentences are (or are not) theorems. **To do** this he must use only the axioms, undefined and defined terms, theorems already proved, and some laws of logic which have been carefully laid down...

Grammar questions

In the passage there are three examples of the pattern **adjective + infinitive**.

Example: It's impossible to do this.

Find the other two.

◆ Note 1

1. Many adjectives can be followed by *infinitives*. This is common when we are talking about feelings and reactions.

Example: I'm sorry to disturb you.
She was very pleased to see me.

2. *Infinitive* can be used in the structure with a preparatory subject "it".

Example: It's difficult to find the answer.
means: To find the answer is difficult.
infinitive phrase = it

PRACTICE

1. *Work in pairs to decide which of the following adjectives can be used in the sentences* "He was ... to see her".

Example: He was happy to see her.

afraid, anxious, nervous, fine, lazy, happy, beautiful, lucky, ready, right, intelligent, surprised, unusual, well, willing, wrong.

2. Rewrite these sentences following the model to make them more natural.

Example: To say no to people is hard → It's hard to say no to people.

a. To select two points on a line, labeling them and referring to the line in this way is more convenient.

→

b. To use colored chalk is more effective.

→

c. To memorize all of these relations is very difficult.

→

d. To distinguish the elements of a set from the “non elements” is very essential.

→

e. To point out that elements of a set need not be individual, but may themselves be sets is very important.

→

f. To determine the exact image in that case is impossible.

→

g. To have a more simplified system of notation is desirable.

→

h. To see that the meaning of an expression, depending on its context, is very clear.

→

i. To give your full name is compulsory.

→

j. To do the measuring as accurately as possible is very necessary.

→

k. To cut \overline{MN} in two or three parts is permissible.

→

3.1 Make sentences using the ideas given:

- your writing / impossible / read
- useful / use / heating pad
- necessary / read carefully / theorem / beforehand
- silly / get upset / small things
- equation / difficult / solve
- important / drink / lots of liquids.
- unfair / criticise him

3.2 Continue these sentences using ideas in example 3.1.

Example: I like John very much. It's very interesting to talk to him.

a. *I must ask my teacher because*

-
- b. You shouldn't be annoyed about that.
 - c. You write awfully.
 - d. I think he wasn't intentional.
 - e. I've got a temperature
 - f. Don't hurry. You should look at the theorem first,
 - g. What should you do for a backache? –

PRESENTATION 2

Read the sentences.

To solve this equation multiply each term in it by the quantity that proceeds it. The important step in solving such a problem is to read the problem carefully to understand it correctly. In order to leave the number unchanged in value we multiply it by the same power of ten

Grammar questions

Answer the following questions.

1. Why must we multiply each term in the equation by the quantity proceeding it?
2. Why must we multiply the number by the same power of ten?

In the sentences above, the *infinitive* is used to express purpose. Read the passage in the *presentation 1* again and find some more examples of the *infinitive of purpose*.

◆ **Note 2**

+ We can use “to–infinitive” to say why somebody does something.

Example: To check the result of addition you have to subtract this number from the sum obtained.

+ In order to ..., so as to ... are common before be, know, have and before other verbs in a formal style.

Example: – She studied English in order to have a better job.

– I came to Britain so as to know more about British culture.

PRACTICE

1. Grammar

1.1 Change the following sentences using to–infinitive for purpose.

Example: We have to subtract this number from the sum obtained because we want to check the result of addition.

→ To check the result of addition, we have to subtract this number from the sum obtained.

a. We must know the details because we want to understand the situation.

→

b. You must do the following because you want to operate this machine.

→

- c. He put the figures in a table because he wants to look at the data.
 →
- d. He included the empty set at the beginning because he wants to have a complete table.
 →
- e. We made a conjecture and then proved this because we want to have the correct procedure.
 →

1.2 Match a line in A with a line in B.

A	B
a. We apply the Euclidean algorithm	to denote sets.
b. We use the symbol \in	let us use the unit circle.
c. We use the braces { }	to mean “ is an element of ”.
d. To clarify this idea	we return to one-dimensional geometry and line segments.
e. To fix our thoughts	we must find a statement that conforms to the rule stated above.
f. We draw a picture	to express GCD as a linear combination.
g. To find the negation of some statements,	to show the physical realization on this vector sum.
h. In order to introduce the concept of measure,	we present some examples of set.

2. Speaking

Work in pairs

2.1 Ask and answer the questions using phrases in the box.

- understand the situation
- know the truth
- know which of two unequal numbers is larger
- check the result of addition
- hear everybody’s viewpoint

- a. Why do you have to subtract this number from the sum obtained?
 b. Why do you need a more exact description?

- c. Why must you make sure that you have considered every detail?
- d. Why must one know the detail?
- e. Why do you question everyone?

2.2 Complete the following sentences.

- a. In order to speak English well
- b. In order to communicate effectively
- c. In order to find the difference
- d. To make sure that everything is correct
- e. To become a good student
- f. To get a scholarship

SKILLS DEVELOPMENT

• **Reading**

1. Pre – reading task

1.1 We know the noun “operation” (such as “the set operation” in the text) is formed by adding suffix *-tion* that has the meaning “the act of”. *Use suffix -tion or -ation to form the nouns from the following verbs:*

interpret	react	locate
communicate	concentrate	represent
relate	multiply	

1.2 State the Cartesian product.

2. Read the text.

THE COORDINATE PLANE

Now we want you to consider two sets: A and B, such that $A = \{ a, b, c \}$ and $B = \{ d, e \}$. We will form a new set from sets A and B, which we will call the Cartesian product, or simply the product set, by forming all possible ordered pairs (x, y) such that x is from set A and y is from set B. This new set is denoted by $A \times B$ (read A cross B).

$$A \times B = \left\{ \begin{array}{l} (a, d), (a, e) \\ (b, d), (b, e) \\ (c, d), (c, e) \end{array} \right\}$$

Let us use the notation $n(A)$ to mean the number of elements in set A and $n(A \times B)$ to mean the number of elements (ordered pairs) in $A \times B$. Observe that $n(A \times B) = 6$ and that $n(A) = 3$ and $n(B) = 2$. Since $3 \times 2 = 6$, we see there is a relationship of some importance between the set operation of forming the Cartesian product and multiplication of numbers $n(A) \times n(B) = n(A \times B)$. Now let us form $B \times A$.

$$B \times A = \left\{ \begin{array}{l} (d,a) , (d,b) , (d,c) \\ (e,a) , (e,b) , (e,c) \end{array} \right\}$$

You may have noticed that no elements (ordered pairs) of $B \times A$ are the same as those of $A \times B$, though their numbers are still the same. This means that $A \times B \neq B \times A$, while $n(A) \times n(B) = n(B) \times n(A)$. Forming the product set is a non-commutative operation. In this case it is a non-commutative multiplication.

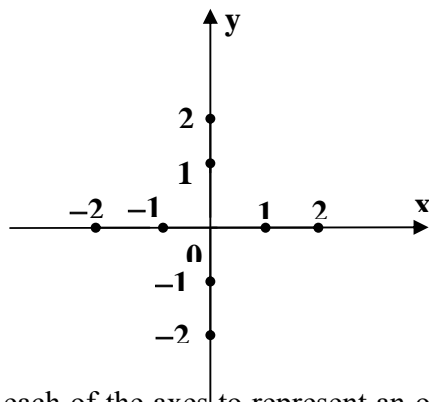
In our next step we do something that at first will seem purposeless. Given that set $A = \{ a, b, c \}$, we will form the new set $A \times A$.

$$A \times A = \left\{ \begin{array}{l} (a,a) , (a,b) , (a,c) \\ (b,a) , (b,b) , (b,c) \\ (c,a) , (c,b) , (c,c) \end{array} \right\}$$

This is, of course, the Cartesian product of set A with itself, and you will wonder what you can do with it. Its use will become clear if we let $X = \{ 0, 1, 2 \}$ and let $Y = \{ 0, 1, 2 \}$. Then find $X \times Y$ the Cartesian product of a set with itself since $X = Y$.

$$X \times Y = \left\{ \begin{array}{l} (0,0) , (0,1) , (0,2) \\ (1,0) , (1,1) , (1,2) \\ (2,0) , (2,1) , (2,2) \end{array} \right\}$$

We then interpret this set of ordered pairs of numbers as a set of points in a plane such that to each point there corresponds one ordered pair of numbers and vice versa. Now it is necessary for us to set up a model for geometric interpretation. To do this we intersect two number lines at the zero point, or origin of the graph, so that the lines are perpendicular to each other. Label the number lines as shown in the following figure by choosing X to denote the set of points on the horizontal line and Y to denote the set of points on the vertical line. Now we assign positive numbers to the right half line of X and negative numbers to the left half line of X . Similarly we assign positive numbers to the upper half line of Y and negative numbers to its lower half line. The two number lines are called axes. We speak of the x axis when we refer to the horizontal number line and of the y axis when we refer to the vertical number line. We now have an interpretation such that every ordered pair of numbers labels a point in the plane determined by the X and Y axes.



Since we find each of the axes to represent an ordered set of points and both axes to cooperate in determining the plane, such a system is said to be a coordinate system and the plane determined by it is said to be a coordinate plane. Each ordered pair (x, y) tells you how to locate a point in the coordinate plane, by starting from the origin. (x, y) means: first move x units from $(0, 0)$ along the x axis to the right or left (indicated by + or

– preceding the first numeral of the pair); then move y units from that point parallel to the y axis (up or down as indicated by $+$ or $-$ preceding the second numeral of the pair).

[

Comprehension check

1. Answer the following questions.

- What are the two number lines as we have used them for the coordinate systems called?
- What is the horizontal number line often referred to?
- What is the vertical number line often referred to?
- Into how many parts do the two axes of the coordinate system divide the plane?
- If both coordinates of a point are 0, where is the point located?
- What does a coordinate of a point tell you?
- What does each of the axes represent?

2. Say if these statements are True (T) or False (F).

- Each ordered pair (x, y) tells you how to locate every point in the plane.
- We know each of the numbers of a pair to be either positive or negative.
- The operation of forming the Cartesian product is commutative.
- To every ordered pair of real numbers there correspond several points on the plane.
- There is no one to one correspondence between real numbers and the points on a line.

• Listening

1. Here are some words phrases related to analytic geometry

abscissa (first coordinate, x coordinate)

graph

cartesian coordinate plane

ordered pairs of coordinates

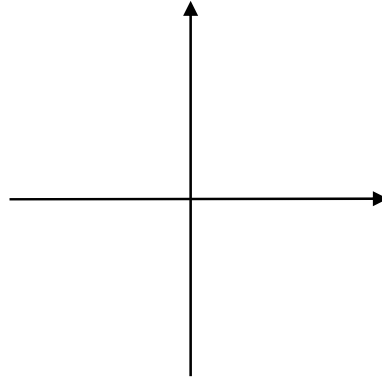
ordinate (second coordinate, y coordinate)

origin

2. Listen to the tape. Try to fill the terms above.

- A plane in which two perpendicular axes have been constructed is called.....
- The point of intersection of these two lines is called
- The ordered pair (a, b) corresponding to point A in that plane is called.....
- a is called
- b is called
- The point A is called

3. *Listen again. Complete the figure and label it.*



4. **Speaking**

Work in pairs to explain the terms above using the figure.

TRANSLATION

Translate into Vietnamese.

1. The Cartesian product named after René Descartes (1596 – 1650), a French mathematician and philosopher who first wrote about analytic geometry – a “union” of algebra and geometry. The Latin equivalent of Descartes is Cartesius and this was the name he used since Latin was the universal scientific language of his day.
2. George Cantor (1845 – 1918) in the years 1871 – 1884 created a completely new and very special mathematical discipline, the theory of sets, in which was founded a theory of infinity with all the incisiveness of modern mathematics.

Just for fun ☺ Ask and answer

- A. asks: – Which animal has four legs but can swim and fly?
B. answers: – Two ducks!

UNIT 8

THE PASSIVE

PRESENTATION

1. Complete the following sentences with the verb “to be” in the correct tense.

- a. Sir Isaac Newton a very famous mathematician and physicist and his devotions to mathematics very important.
- b. In the 1660s he in Grantham and his half-brothers and half-sisters at home in Lincolnshire.
- c. Mr. Forester to Paris for a long time.
- d. He in Paris next week.

2. Write in the Past Simple and the Past Participle of the following verbs.

Some are regular and some irregular.

Bare infinitive	Past Simple	Past Participle
take	took	taken
treat
invent
think
relate
go
read
submerge
arithmetize

3. Read the following paragraph and answer the questions.

RATIO AND PROPORTION

A ratio is an indicated division. It should be thought of as a fraction. The language used is: “the ratio of a to b ” which means $a \div b$ or $\frac{a}{b}$ and the symbol is $a : b$. In this notation a is the first term or the antecedent, and b is the second term or the consequent. It is important to remember that we treat the ratio as a fraction. A proportion is a statement that two ratios are equal. Symbolically we write: $a : b = c : d$ or $\frac{a}{b} = \frac{c}{d}$.

The statement is read “ a is to b as c is to d ” and we call a and d the extremes, b and c the means, and d the fourth proportional. Proportions are treated as equations involving fractions. We may perform all the operations on them that we do on

equations, and many of the resulting properties may already have been met in geometry.

Questions:

- a. What should a ratio be thought of as?
- b. How is the statement read when we write $a : b = c : d$ or $\frac{a}{b} = \frac{c}{d}$?
- c. How are proportions treated?

4. Grammar questions

“The statement is read “*a is to b as c is to d*””.

- Is it important who reads the statement?
- What is the main interest of the sentence?

◆ **Note**

- + Passives are very common in technical writing where we are more interested in facts, processes, and events than in people.
- Data is transferred from the internal memory to the arithmetic logical unit.
- Distributed systems are built using networked computers.
(→ about facts / processes)
- The organization was created to promote the use of computers in education.
(→ about event)
- + With the passives, we can use by + noun if we need to show who / what is responsible for the facts / processes or events.
- A new method for studying geometric figures and curves, both familiar and new were created by Descartes and Fermat.

PRACTICE

1. Writing

In the columns below, write in the passive verb forms from the text.

Present Simple	Past Simple	Present Perfect	Will Future
.....	will be thought
is used
is read
are treated
.....	have been met

Complete the rule:

The passive is formed with the auxiliary verb + the

2. Grammar

2.1 Fill in the gaps using the correct form of the verb in brackets.

All calls (register) by the Help Desk staff.
Each call (evaluate) and then (allocate) to the relevant support group. If a visit (require), the user (contact) by telephone, and an appointment(arrange). Most calls (deal with) within one working day. In the event of a major problem requiring the removal of a user's PC, a replacement can usually (supply).

2.2 Make the sentences passive. Use "by ..." only if it is necessary to say who does / did the action.

- a. Charles Babbage designed a machine which became the basis for building today's computer in the early 1800s.
- b. People submerged geometry in a sea of formulas and banished its spirit for more than 150 years.
- c. People often appreciate analytical geometry as the logical basis for mechanics and physics.
- d. Bill Gates founded Microsoft.
- e. People call the part of the processor which controls data transfers between the various input and output devices the central processing unit (CPU).
- f. You may use ten digits of the Hindu–Arabic system in various combinations. Thus we will use 1, 2 and 3 to write 123, 132, 213 and so on.
- g. Mathematicians refer to a system with which one coordinates numbers and points as a coordinate system or frame of reference.
- h. People similarly establish a correspondence between the algebraic and analytic properties of the equation $f(x, y) = 0$, and geometric properties of the associated curve.
- i. In 1946 the University of Pennsylvania built the first digital computer.

2.3 Change the following passive sentences into active.

- a. This frame of reference will be used to locate a point in space.
- b. Although solid analytic geometry was mentioned by R.Descartes, it was not elaborated thoroughly and exhaustively by him.
- c. Most uses of computers in language education can be described as CALL.
- d. Since many students are considerably more able as algebraists than as geometers, analytic geometry can be described as the "royal road" in geometry that Euclid thought did not exist.
- e. Now new technologies are being developed to make even better machines.
- f. Logarithm tables, calculus, and the basis for the modern slide rule were not invented during the twentieth century.
- g. After World War 2 ended, the transistor was developed by Bell Laboratories.
- h. The whole subject matter of analytic geometry was well advanced, beyond its elementary stages, by L.Euler.

- i. In general, fields may be required from several logical tables of data held in a database.

SKILLS DEVELOPMENT

- **Reading and speaking**

1. Pre – reading task

1.1 Use your dictionary to check the meaning of the following words.

ellipse, hyperbola, parabola.

1.2 Give the definitions for each of words in exercise 1.1.

- a. A type of cone that has an eccentricity equal to 1. It is an open curve symmetrical about a line.
- b. A type of cone that has an eccentricity between 0 and 1 ($0 < e < 1$). It is a closed symmetrical curve like an elongated circle—the higher the eccentricity, the greater the elongation.
- c. A type of cone that has an eccentricity (e) greater than 1. It is an open curve with two symmetrical branches.

2. Read the text.

The evolution of our present-day meanings of the terms “ellipse”, “hyperbola”, and “parabola” may be understood by studying the discoveries of history’s great mathematicians. As with many other words now in use, the original application was different from the modern.

Pythagoras (c.540 B.C.), or members of his society, first used these terms in connection with a method called the “application of areas”. In the course of the solution (often a geometric solution of what is equivalent to a quadratic equation) one of three things happens: the base of the constructed figure either falls short of, exceeds, or fits the length of a given segment. (Actually, additional restrictions were imposed on certain of the geometric figures involved.) These three conditions were designated as ellipsis “defect”, hyperbola “excess” and parabola “a placing beside”. It should be noted that the Pythagoreans were not using these terms in reference to conic sections.

In the history of conic sections, Menaechmus (350 B.C.), a pupil of Eudoxus, is credited with the first treatment of conic sections. Menaechmus was led to the discovery of the curves of conic sections by a consideration of sections of geometrical solids. Proclus in his summary reported that the three curves were discovered by Menaechmus; consequently, they were called the “Menaechmian triads”. It is thought that Menaechmus discovered the curves now known as the ellipse, parabola and hyperbola by cutting cones with planes perpendicular to an element and with the vertex angle of the cone being acute, right or obtuse respectively.

The fame of Apollonius (c.225 B.C.) rests mainly on his extraordinary conic sections. This work was written in eight books, seven of which are presented. The work of Apollonius on conic sections differed from that of his predecessors in that he obtained all of the conic sections from one right double cone by varying the angle at which the intersecting plane cuts the element.

All of Apollonius's work was presented in regular geometric form, without the aid of algebraic notation of present day analytical geometry. However, his work can be described more easily by using modern terminology and symbolism. If the cone is referred to a rectangular coordinate system in the usual manner with point A as the origin and with (x, y) as coordinates of any points P on the cone, the standard equation of the parabola $y^2 = px$ (where p is the length of the latus rectum, i.e. the length of the chord that passes through a focus of the conic perpendicular to the principal axis) is immediately verified. Similarly, if the ellipse or hyperbola is referred to a coordinate system with vertex at the origin, it can be shown that $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ or $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, respectively. The three adjectives "hyperbolic", "parabolic", and "elliptic" are encountered in many places in maths, including projective geometry and non-Euclidean geometries. Often they are associated with the existence of exactly two, one, or more of something of particular relevance. The relationship arises from the fact that the number of points in common with the so called line at infinity in the plane for the hyperbola, parabola and ellipse is two, one and zero respectively.

Comprehension check

1. Answer the following questions.

- What did the words "ellipse", "hyperbola" and "parabola" mean at the outset?
- What did these terms designate?
- Who discovered conic sections?
- What led Menaechmus to discover conic section?
- What were the curves discovered by Menaechmus called?
- What did Menaechmus do to obtain the curves?
- Who supplied the terms "ellipse", "parabola", "hyperbola" referring to conic sections?

2. Choose a suitable heading for the text.

- The discoveries of history's great mathematicians.
- Menaechmus discovered three curves of conic sections by a consideration of sections of geometrical solids.
- History of the terms "ellipse", "hyperbola" and "parabola".

• Listening

How can the use of computers help the teaching of maths? To answer this question, you will hear the text about "Algorithms". Try to listen and fill in the gaps using the words from the box

- basic tool (n)
- concern (v)
- depend (v)
- unsolvability (n)
- fundamental notion (n)
- number (n)
- organization (n)
- represent (v)
- representation (n)
- show (v)

Algorithms

Originally algorithms (1) solely with numerical calculations; Euclid’s algorithms for finding the greatest common divisor of (2) – is the best illustration. There are many properties of Euclid’s powerful algorithm which has become a(3) in modern algebra and number theory. Nowadays the concept of an algorithm is one of the most (4) in maths. Experience with computers (5) that the data manipulated by programs can represent virtually anything. In all branches of maths, the task to prove the solvability or (6) of any problem requires a precise algorithm. In computer science the emphasis has now shifted to the study of various structures by which information (7) and to the branching or decision making aspects of algorithms, which allow them to fall on one or another sequence of the operation (8) on the state of affairs at the time. It is precisely these features of algorithms that sometimes make algorithms models more suitable than traditional maths models for the (9) and (10) of knowledge.

TRANSLATION

• *Translate into Vietnamese.*

1. The set of points which are equidistant from a fixed point and a fixed line is a parabola. The fixed point is the focus of the parabola and the fixed line is its directrix.
2. The set of points, the sum of whose distances from two fixed points is a constant, is an ellipse. The fixed points are the foci of the ellipse and the constant is the length of its major diameter.
3. The set of points, the differences of whose distances from two fixed points is a constant, is a hyperbola. The fixed points are the foci of the hyperbola and the constant is the length of its transverse axis.

• *Translate into English.*

1. Tỷ số của hai số thực a, b là thương số của a và b . Tỷ số của a với b được viết là $a \div b$ hoặc $a : b$ hoặc a / b .
2. Số hữu tỷ ba phần tư được viết ở dạng phân số là $3 / 4$, ở dạng thập phân là 0.75.
3. Trong hình học, tỷ số của a và b thường là tỷ số đo đạc của hai đại lượng cùng đơn vị.
4. Tỷ lệ thức là một mệnh đề có hai (hoặc hơn) tỷ số bằng nhau. Ví dụ: $\frac{a}{b} = \frac{c}{d}$ là một tỷ lệ thức. Nó cũng có thể viết là $a : b = c : d$.

5. Số hạng thứ nhất và số hạng thứ tư (trong trường hợp này là a và d) là các cực trị của tỷ lệ thức; số hạng thứ hai và số hạng thứ ba (trong trường hợp này là b và c) gọi là hai số hạng ở giữa của tỷ lệ thức.

UNIT 9

RELATIVE CLAUSES

PRESENTATION

1. *Do you know what an electronic computer is?*
Here is a definition of it:
“An electronic computer is a device that can accept information, store it, process it and present the processed results in some acceptable form.”

2. *Read the passage below. Use a dictionary to check vocabulary where necessary.*

WHAT IS AN ELECTRONIC COMPUTER?

A most important adjunct to this definition is that a computer is told how to process the information by instructions, which are stored in coded form inside the computer. A computer thus differs radically from a calculator, which can do the same thing that a computer does, except that the instructions are not stored inside the machine. The coded instructions are called a program.

Any computer or calculator contains devices for five main functions: input, storage, arithmetic, control and output. Input refers to the process by which information is put into a machine. Output is the process by which the results are moved out of the machine. Storage refers to the mechanism that can retain information during calculation and furnish it as needed to other parts of the machine. The arithmetic unit is that part of the machine, which can carry out one or more of the basic arithmetic operations on the information held in storage. Finally, the control refers to those parts of the machine that dictate the functions to be performed by all the others parts.

The main difference between computers and calculators is that the instructions telling the computer what to do must be placed in storage before the computer proceeds with the solution of a problem. These instructions, which are made up of ordinary decimal digits are placed in the same storage device that holds the data.

3. Grammar questions

3.1 Read these sentences.

- A computer differs radically from a calculator, which can do the same thing that a computer does.
- Input refers to the process by which information is put into the machine.

- c. Output is the process by which the results are moved out of the machine.
- d. Storage refers to the mechanism that can retain information during calculation and furnish it as needed to others parts of the machine.
- e. These instructions, which are made up of ordinary decimal digits are placed in the same storage device that holds the data.

3.2 Answer the following questions.

- a. What are the underlined clauses called?
- b. What is the role of the relative pronouns “which”, “that” in the relative clauses above?
- c. Why are relative clauses used?

PRACTICE

1. Grammar

1.1 Join the following sentences together using *who, that, whose, which,*

where.

- a. The equation of any curve is an algebraic equality. This equality is satisfied by the coordinates of all points on the curve but not the coordinates of any other point.
- b. A programmer is a person. He prepares programs to solve problems.
- c. The arithmetic logical unit is a part of the CPU. Arithmetic and decision making operations are done in it.
- d. A function is a set of ordered pairs. Its first elements are all different.
- e. The part of the processor is called the control unit. The processor controls data and transfers it between the various input and output devices.
- f. A window is an area of the computer screen. You can see the contents of a folder, a file or a program in it.
- g. Leonard Euler first gave examples of long analytical procedures. Conditions of the problem are first expressed by algebraic symbols and then pure calculation resolves the difficulties.
- h. Their new range of cosmetics will be launched next month. They have spent £10 million on it.

◆ Note 1

+ It is possible, particularly in formal or written language, to put prepositions like to, from, about, on, etc. in front of relative pronouns.

Example: The woman to whom I spoke was extremely helpful.

+ It is much more common to put words like to, from, about, on, etc. at the end of relative clauses.

Example: The woman (that) I spoke to was extremely helpful.

1.2 Rewrite the sentences putting the prepositions in front of or at the end of the relative clauses.

- a. She works for a company. It has a very good reputation.
The company.....
- b. After the great impetus given to the subject by R.Descarter and P.Fermat, we find analytical geometry in a form. With the form we are familiar today.
After the great impetus given to the subject by these two men,
- c. Most systems have a special area of the screen. On the screen icons appear. Most systems have a special area of the screen
- d. The salesperson was correct in saying that goods must be returned to the store. From there they were purchased.
The salesperson was correct in saying that goods must be returned to the store
- e. I deal with customers. Most of them are very pleasant.
Most of the customers.....
- f. The simplest problem of tracing polar curves is the case. There is only one value of θ in this case.
The simplest problem.....

◆ Note 2

We often use the relative clauses to give definitions / explanations.

Example:

A computer is an electronic device *which / that* processes information.

A	B	C
A WAN	is a device	that connects over long – distance telephone lines.
A modem	is a surface generated by the motion of a straight line	that remains at a constant distance, the radius, from a fixed point, the center.
A plane	is the locus of a moving point	that consists of a closed series of arcs of great circles; no arc must exceed a half of a great circle.
The compiler's operating system	is a quantity	which always passes through a fixed point and intersects a given line.
A conical surface	is a figure	which serves a dual purpose because it acts as a Modulator and a Demodulator.
A spherical polygon	is a true system program	which has magnitude and direction.
A vector	is a network	that a straight line joining any two points of the surface lies entirely in the surface.
A sphere	is a surface	which control the central processing unit (CPU), the input, the output, and the secondary memory devices.

2. Vocabulary

Match a line in A with a line in B and then use the relative clauses in

C to give some definitions.

SKILLS DEVELOPMENT

- **Reading and speaking**

1. Pre – reading task

Look at these circles and let's perform an experiment.



Here are nine circles. Five are black, four are white. If you were told to cover one circle with your finger, you might choose any one of the nine. But you are more likely to choose a black circle than a white, because there are more black circles than white ones. Indeed, the probability that you will cover a black circle is $\frac{5}{9}$, the ratio of the number of black circles to the total number of circles.

2. To answer the question “What is the probability in maths? ”, we will read the text about it.

PROBABILITY OF OCCURENCE

In mathematical language the choice, the probability of success is the ratio of the number of ways in which the trial can succeed to the total number of ways in which the trial can result. Here nothing favors the choice of any particular circle; they are all on the same page, and you are just as likely to cover one as another. The trial can result in five ways; there are five black circles. The trial can result in nine ways; there are nine circles in all (in exercise 1.1). If p represents the probability of success, then $p = \frac{5}{9}$.

Similarly, the probability of failure is the ratio of the number of ways in which the trial can fail to the total number of ways in which it can result. If q represents the probability of failure, in this case $q = \frac{4}{9}$. Notice that the sum of probability of success and failure is 1. If you put your finger on a circle, it is certain to be either a black circle or a white one, for no other kind of circle is present. Thus $p + q = \frac{5}{9} + \frac{4}{9} = 1$. The probability that an event will occur can not be more than 1. When $p = 1$, success is a certainty. When $q = 1$, failure is sure.

Let S represent the number of ways in which a trial can succeed. And let f represent the number of ways in which a trial can fail.

$$p = \frac{S}{S+f} ; q = \frac{f}{S+f} ; p+q = \frac{S}{S+f} + \frac{f}{S+f} = 1$$

When S is greater than f , the odds are S to f in favor of success, thus the odds in favor of covering a black circle are 5 to 4. Similarly, when f is greater than S , the odds are f to S against success. And when S and f are equal, the chances are even; success and failure are equally likely. Tossing a coin illustrates a case in which S and f are equal. There are two sides to a coin, and there is no reason why a normal coin should fall one side up rather than the other. So if you toss a coin and call heads, the probability that it will fall heads is $\frac{1}{2}$. Suppose you toss a coin a hundred times, for each of the hundred trials, the probability that the coin will come down heads is $\frac{1}{2}$.

You might expect fifty of the tosses to be heads. Of course, you may not get fifty heads. But the more times you toss a coin, the closer you come to the realization of what you expect.

If p is the probability of success on one trial, and K is the number of trials, then the expected number is Kp . Mathematical expectation in this case is defined as Kp .

Comprehension check

1. Answer the following questions.

- a. What does the article deal with?
- b. If you were shown 9 red circles and 6 black circles and were asked to choose one of them which on these circles would you be likely to choose? Why?
- c. Can you give the definition of the probability of failure? What is it?
- d. What are the odds in case $f > S$?
- e. What are the odds in case $f < S$?
- f. Suppose $S = f$, what would the chances be?
- g. Could you give some examples to illustrate a case when S and f are equal?

2. Are these statements true or false? Correct the false statements.

- a. The trial can succeed in nine ways when you suppose that you have nine circles.
- b. The sum of the probability of success and failure is equal to 1.
- c. The probability that an event will occur can be more than 1.
- d. In tossing two coins the fact that one fell heads would not affect the way the other fell.

3. Fill in each gap using a word from the text.

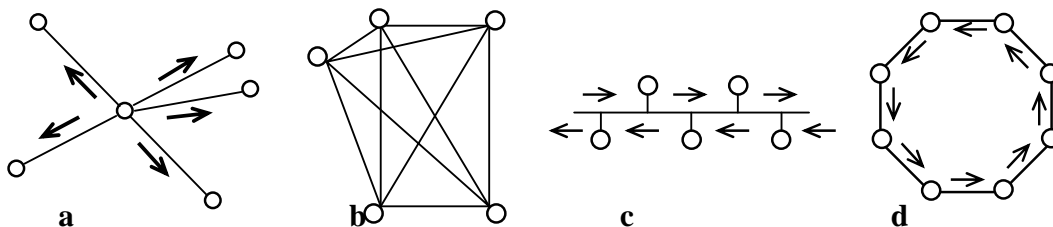
- a. There are differences of opinion among mathematicians and philoso- phers about theory.
- b. Suppose two dice are thrown. What are the chances that the of the faces is five?
- c. Two coins are simultaneous. Since a coin will come down () or tail (T), each possible outcome is a member of $A \times A$ where $A = \{ \dots, T \}$.
- d. To describe this sample space each situation in terms of events and discuss the chances of each event
- e. When we try to do something several times we say that we have had several

• **Listening and speaking**

1. Pre – listening task

Study these diagrams. They show four network topologies. *Try to match each diagram with the correct name.*

1. ring 2. bus 3. star 4. mesh



2. Listen to the tape and check your answers. *The recording describes three topologies.*

3. Which topologies do these statements refer to?

- a. has a server computer at the centre and a separate cable which connects the server to each of the other computers in the network.
- b. In , each computer is connected to its neighbour in a circle. The data flows in one direction round the ring.
- c. has all the computers that connect to a common cable.

TRANSLATION

Translate into Vietnamese.

The fundamental law of probability:

If a certain event can occur in n_1 different ways, and if, after it has happened in one of these, another event can occur in n_2 different ways, then the ordered pair of events can occur in $n_1.n_2$ different ways.

Just for fun

☺ **THE ARITHMETIC PROBLEM**

Teacher: – “ In a family, there are five children and the mother has only four potatoes to divide among them. She wants to give each child an equal share. What does she have to do? ”

A pupil: – “ She has to mash the potatoes, teacher! ”

UNIT 10

CONDITIONAL SENTENCES – FIRST AND ZERO

PRESENTATION

1. Answer the following questions.

- What is a geometric progression?
- What will the sum of the first six terms of the G.P. be?

2. Read the text below to find the answers.

SEQUENCES OBTAINED BY REPEATED MULTIPLICATION

A geometric progression (G.P.) is a sequence of numbers obtained by repeated multiplication. If a, b and c are three numbers in a G.P., there is $\frac{b}{a} = \frac{c}{b}$. Consider the first three terms of a geometric sequence. Let a represent the first term, and let r represent the common ratio.

First term	:	$a = ar^0$
Second term	:	$a.r^1$
Third term	:	$a.r.r = a.r^2$

For each term, the number of times r is used as a multiplier is 1 less than the number of the term. If the total number of terms in a G.P. are n then to find the n-th or last term, r will have to be used as a multiplier (n – 1) times. That is, $b_n = ar^{n-1}$. On the chessboard G.P. 1, 2, 4, 8, ..., the value of a is 1 and r is 2. Since there are 64 squares on a chessboard, n = 64. Then $b_{64} = 1.2^{64-1}$ or accordingly, $b_n = 2^{63}$. You can readily find the value of b_{64} by making use of logarithms; in standard form it is about 9.2×10^{18} . The chessboard G.P. is clearly understood to be an increasing progression. G.P. with a positive first term in which the common ratio is a number less than 1 is said to be a decreasing sequence. The common ratio may be negative. If this is the case and the terms are alternatively positive and negative as in +1, -2, +4, -8, +16, ... the sequence will move back and forth or oscillate from positive to negative, or from negative to positive. Such a G.P. is an oscillating sequence. The formula for the last term in a G.P. can, like any formula, be evaluated for any letter in it. If you wish to find the value of a, it will be convenient to apply the formula in the form $a = \frac{b_n}{r^{n-1}}$. If you want to find the value of r or of n, it will be well to apply it in the form $r^{n-1} = \frac{b_n}{a}$. Logarithms may prove helpful, or else, you may be able to apply the laws of exponents.

3. Work in pairs to discuss the grammar questions.

- a. If a , b , and c are three numbers in a G.P., there is $\frac{b}{a} = \frac{c}{b}$.
- b. If the terms are positive and negative, the sequence will move back and forth from positive to negative or from negative to positive.
- c. If you want to find the value of r or of n , it will be well to apply it in the form $r^{n-1} = \frac{b_n}{a}$.

• **Grammar questions**

- What sentences is used to express a possible condition and a probable result?
- What sentence is used to express condition that are always true with automatic or habitual results?
- What is the difference between sentences a, b and c?

Complete these rules.

- a. The zero conditional:

if – clause

main clause

IF +.....

,

- b. The first conditional:

if – clause

main clause

IF +.....

, WILL

PRACTICE

• **Speaking**

1. Work in pairs to make a dialogue following the model:

- A: – What will you do if you draw a straight line? (to subtend the angle)
- B: – If you draw a straight line, you will subtend the angle.
- a. What will you do if you divide 25 by 4? (the remainder be equal to 1)
- b. What will they do if they follow the rule? (to find the solution)
- c. What will you need if you want to play back anything from your computer on a TV monitor? (to need a print-to-tape device)
- d. What will your computer system have if it follows the American TV standard? (to have a vertical refresh rate at 60 KHz)

2. In a sentence with an *if-c* clause we can use the imperative, or other modal verb such as: *may, can, must*, etc., instead of *will + infinitive*.

Choose the best option from the words in brackets to complete the following dialogue.

Peter: I'll be at the meeting this afternoon. So if Hans⁽¹⁾ (will call / calls⁽²⁾ / tell/ you'll tell) him I'll give him a ring later.

Barton: Ok, but there's one other thing. You've got a meeting with Mr.Pierre at 5.00. Will you be back by then?

Peter: It depends, really, but I'll call you. If the meeting⁽³⁾ (will go on / goes on) after 4.30, you⁽⁴⁾ (will / can) cancel my appointment with Mr.Pierre. But if it has already finished by then, I⁽⁵⁾ (may / can) be able to get back in time.

Barton: Anything else?

Peter: Yes, if you⁽⁶⁾ (will manage / manages) to get hold of Kevin, you⁽⁷⁾ (must / will) get the October sales figures from him. I need them today. The Chairman⁽⁸⁾ (may come / can come) to the sales meeting tomorrow, and if he does, he⁽⁹⁾ (is going to want / must want) to see them.

• **Grammar**

1. **Complete the sentences with the words below. Are the sentences first (F) or zero (Z) conditional?**

- a. If you your screen for too long, you a headache.
- b. If the market for portable computers, prices even more next year.
- c. If the number n a composite number, the ring Z_n zero divisors.
- d. If you your VDU in direct sunlight, it damaged.
- e. If the field P in a greater field \bar{P} , the ring $P[x]$ a subring of the ring $\bar{P}[x]$.
- f. If you pirated software, it is unlikely that you a problem with computer viruses.

grows	will get	do not copy	possess
will be	will be reduced	be	leave
look at	be contained	will have	

2. **Complete these sentences.**

- a. If I have free time this weekend,
- b. If I go on holiday this year,
- c. If I carry on learning English,
- d. If I feel tired this afternoon,
- e. If I stay in my future job – teacher of maths,

3. Match the if – clauses to the main clauses to make complete sentences.

- | | |
|---|---|
| 1. If you have a modem, | a. we will lose all our latest data. |
| 2. If you never back up your hard disk, | b. you will miss important new products. |
| 3. If the characteristic of a field is equal to p , | c. the ring Z_n is a field. |
| 4. If you never read computer magazines, | d. you will be able to access our bulletin board. |
| 5. If the number n is prime, | e. you will probably lose some important files. |
| 6. If the system crashes, | f. then for any element a of the field we'll have the equality $pa = 0$. |

SKILLS DEVELOPMENT

● **Reading**

1. Pre – reading task

Find the correct definition of topology.

- Topology
- a) is the study of those properties of geometrical figures that are invariant under continuous deformation.
 - b) is that branch of geometry which deals with those properties of figures which are changed by continual deformation.
 - c) is the study of those properties of “topological spaces” that are variant under “homomorphism”.

2. Read the text below.

TOPOLOGY

We know that modern maths is composed of many different divisions. Despite its rigorousness topology is one of the most appealing. Its study is today one of the largest and most important of maths activities. Although the study of polyhedra held a central place in Greek geometry, it remained for Descartes and Euler to discover the following fact: In a simple polyhedra let V denote the number of vertices, E the number of edges, and F the number of faces; then always

$$V + F - E = 2$$

By a polyhedron is meant a solid, whose face consists of a number of polygonal faces. In the case of regular solids all the polygons are congruent and all the

angles at vertices are equal. A polyhedron is simple if there are no “holes” in it, so that its surface can be deformed continuously into the surface of a sphere. There are of course, simple polyhedra which are not regular and polyhedra which are not simple. It is not difficult to check the fact that Euler’s formula holds for simple polyhedra, but does not hold for non simple polyhedra.

We must recall that elementary geometry deals with magnitudes (lengths, angles and areas) that are unchanged by the rigid motions, while projective geometry deals with the concepts (point, line, incidence, and cross – ratio), which are unchanged by the still larger group of projective transformations. But the rigid motions and the projections are both very special cases of what are called topological transformation: a topological transformation of one geometrical figure A into another figure A' is given by any correspondence $P \leftrightarrow P'$ between the points P of A and the points P' of A' , which has the following two properties:

1. The correspondence is biunique. This means to imply that to each point P of A corresponds just one point P' of A' and conversely.
2. The correspondence is continuous in both directions. This means that if we take any two point P, Q of A and move P so that the distance between it and Q approaches zero (0), the distance between the corresponding points P', Q' of A' will also approach zero, and conversely.

The most intuitive examples of general topological transformation are deformations. Imagine, a figure such as a sphere or a triangle to be made from, or drawn upon, a thin sheet of rubber, which is then stretched and twisted in any manner without tearing it and without bringing distinct points into actual coincidence. The final position of the figure will then be a topological image of the original. A triangle can be deformed into any other triangle or into a circle or an ellipse, and hence these figures have exactly the same topological properties. But one cannot deform a circle into a line segment, nor the surface of a sphere into the surface of an inner tube. The general concept of topological transformation is wider than the concept of deformation. For example, if a figure is cut during a deformation and the edges of the cut sewn together after the deformation in exactly the same way as before, the process still defines a topological transformation of the original figure although it is not a deformation. Topological properties (such as are given by Euler’s theorem) are of the greatest interest and importance in many math investigations. There are, in a sense, the deepest and most fundamental of all geometrical properties, since they persist (continue to hold) under the most drastic changes of shape. On the basis of Euler’s formula it is easy to show that there are no more than five regular polyhedra.

Comprehension check

1. Are the statements True (T) or False (F)? Correct the false ones.

- a. All the angles at vertices of regular polyhedra are even.
- b. Any simple polyhedra is regular.
- c. Euler’s formula holds for any kind of polyhedra.
- d. Between the points P of A and the points P' of A' the correspondence is one to one.

- e. The correspondence is not interrupted in one direction.
- f. Using Euler's formula we can illustrate a lot of regular polyhedra.

2. Answer the following questions.

- a. Who first discovered the formula $V + F - E = 2$ for a simple polyhedron?
- b. Which solid could be a polyhedron?
- c. What happens to the polygons if the polyhedron is regular?
- d. Are the magnitudes and projective transformation unchanged in the same cases?
- e. What kinds of polyhedra are there?
- f. Why can a triangle be deformed into any other triangle, into a circle or into an ellipse?
- g. Can we deform a circle into a line segment? Why or why not?
- h. How important are topological properties for mathematicians?

3. Writing

Complete the sentences based on the text.

- a. The most visible examples of general topological transformations are
- b. The general concept of topological transformation is
- c. A polyhedron is simple
- d. The rigid motions and the projections are
- e. In elementary geometry lengths, angles and areas
- f. Projective geometry deals with

4. Vocabulary

The first irregular forms of nouns of Latin and Greek origin.

◆ **Note**

Lots of Latin and Greek original nouns often have plural forms ending in -a, for example, polyhedron → polyhedra

a. Write the plural forms of these nouns.

- | | | | |
|---------------|--------|--------------|--------|
| - continuum | →..... | - medium | →..... |
| - criterion | →..... | - minimum | →..... |
| - curriculum | →..... | - momentum | →..... |
| - datum | →..... | - phenomenon | →..... |
| - equilibrium | →..... | - quantum | →..... |
| - spectrum | →..... | - vacuum | →..... |
| - maximum | →..... | - stratum | →..... |

Complete the rule.

The ending, are changed into in the plural forms.

b. Fill in the gaps with a suitable noun from the list above.

- The notion of four dimensional geometry is a very helpful one in studying physical

- A chord drawn through either focus on the ellipse and perpendicular to the principal axis is called
- All these facts may serve as reference
- Complete surfaces formed with regular polygons such as a complete surface of a cube built up by joining six squares along edges are called.....

• **Listening**

1. We are going to listen to some general principles to follow in debugging a code. Try to understand the meaning of these verb phrases, put them in the right order to debug a code.

- check to see
- cut a tape of the tables
- discard the earlier one
- key the routine
- key the corrections into storage
- set the console error switches.
- think before acting

2. *Listen to the tape. Fill in the gaps and check your answers.*

There are a few fundamental rules in the debugging checklist worth considering:

- a. into storage with test data. In the early stages of the test data need only be in the correct form; the values used are not too important
 - b. that the arithmetic tables are in and are
 - c. the routine keyed in, and the
- This tape is for insurance: you need not read it back at this time.
- d. to stop, etc.
 - e. When you have discovered as many errors as you can, (if your routine has become garbled in storage, first reload the tape you cut at step). Immediately cut a new tape and etc.
 - f. Above all,

TRANSLATION

• *Translate into Vietnamese.*

1. Euler's theorem

The relationship $V - E + F = 2$ for any simple closed polyhedron, where V is the number of vertices, E the number of edges, and F the number of faces. (A simple

closed polyhedron is one that is topologically equivalent to a sphere) The expression $V - E + F = 2$ is called the Euler characteristic, and its value serves to indicate the topological genus.

2. Euler's formula

The formula: $e^{ix} = \cos x + i \sin x$

It was introduced by Euler in 1748, and is used as a method of expressing complex numbers. The special case in which $x = \pi$ leads to the formula $e^{i\pi} = -1$.

- ***Translate into English.***

Một không gian topo là một tập hợp cùng với một cấu trúc cho phép đề cập tới khái niệm hội tụ và liên tục. Phương pháp xây dựng cấu trúc ấy là chỉ ra những tập hợp nào được coi là mở.

UNIT 11

SECOND CONDITIONALS

PRESENTATION

1. Read the text below to find the answers.

UNENDING PROGRESSIONS

If a sequence has a definite number of terms, it is said to be finite, a word meaning limited. But a sequence may have an unending number of terms. For example, the integers of the number system go on forever: if you counted to one billion you could just as well count to one billion and one. You can easily imagine 10^{21} . It is just as possible to imagine the number $10^{21} + 1$. Such a sequence is said to be infinite; it is unlimited.

It is meaningless to ask for the sum of integers in the number system. It would be equally meaningless to ask for the sum of the numbers in any other infinite arithmetic progression (A.P.) if it was an increasing A.P. or decreasing A.P. A G.P. may also be infinite.

Consider the infinite G.P. $1, \frac{1}{2}, \frac{1}{4}, \dots$. You could continue writing the terms of this G.P. indefinitely. If you did so, the number of terms would increase without limit. This fact is shown in symbol form thus: $n \rightarrow \infty$, the arrow meaning approaches, and ∞ being the symbol for infinity. The statement is read “ n approaches infinity” or “tends to infinity”. As $n \rightarrow \infty$, what happens to the sum of the G.P.? To answer this question, notice first that the number of terms increases, the value of the n -th term b_n gets smaller and smaller. It approaches zero; that is, $b_n \rightarrow 0$.

If you add successive terms, you will add a smaller quantity each time.

$$n = 2 : S_n = 1 + \frac{1}{2} = 1\frac{1}{2}$$
$$n = 3 : S_n = 1 + \frac{1}{2} + \frac{1}{4} = 1\frac{3}{4} \dots$$

If you went on like this, you would realize that with each addition, the value of S_n got closer to 2. But no matter how many terms you added, S_n would never reach 2. If it approaches 2; that is, 2 is its limit. As $n \rightarrow \infty$ in the G.P. $1, \frac{1}{2}, \frac{1}{4}, \dots$, $S_n \rightarrow 2$. The limit of S_n is an infinite G.P. is called the sum of the G.P. and is written S_∞ . Thus for the infinite G.P. $1, \frac{1}{2}, \frac{1}{4}, \dots$, $S_\infty = 2$.

2. Grammar questions

2.1 The text contains two types of conditional sentences.

- a. Complete these sentences from the text.
- If a sequence a definite number of terms, it to be finite.
 - If you to one billion, you just as well to one billion and one.
- b. What form of the verb is used:
- in the if clause?
 - in the main clause?
- c. What is each type of conditional called?

2.2 When is each type of conditional used?

- a. Answer the questions.
- Is there a real condition that a sequence has a definite number of terms?
 - Do you want to count to one billion right now?
- b. Which sentence is being talked about:
- an automatic situation?
 - an imaginary situation?

Complete the rule:

Second conditional.

condition		result
IF +	,	WOULD +

◆ Note 1

Second conditional sentences express unreal conditions.

The condition is unreal because:

- a. It is possible in theory but impossible in practice.
- b. It is an impossible speculation.

PRACTICE

1. Speaking

1.1 Look back at the passage above. Work on the following questions in pairs.

- a. In what condition is a sequence of terms said to be finite?
- b. What could happen if you counted to one billion?
- c. What would happen if you continued writing the terms of a G.P. indefinitely?
- d. What does the symbol $n \rightarrow \infty$ mean?

1.2 Imagine what would happen and complete these statements using the main clauses in the box.

would need to install a network
 would not post so many letters each day
 would get a better job

would have a bigger range of typefaces and fonts to choose from

- a. If you wanted to link your PCs with a mainframe,.....
- b. If I knew more programming languages,.....
- c. If we installed a fax machine and e – mail facility,.....
- d. If we bought a better printer,.....

2. Grammar

2.1 Work in pairs to make the dialogue following the model.

A: – What would you do to draw a straight line? (to use a ruler)
 B: – I would use a ruler to draw a straight line.

- a. What would you do to solve this problem? (find the value of the unknown)
- b. What would you suggest for improving the situation? (some modification)
- c. What would you do to be sure of the result? (check the result)
- d. What would you suggest for evaluating this formula? (make use of logarithms)

2.2 Put the verbs in brackets in the correct form.

- a. If we (consider) the third example, we..... (see) that the magnitude of the common ratio was less than 1.
- b. If we (assume) the geometric mean of two numbers to be the square root of their product, what the geometric mean between 2 and 8 (be)?
- c. If the system (crash), we..... (lose) all our latest data.
- d. If paradoxes (be not) so subtle and colourful by Zeno’s, mathematicians (not pay attention) to them.
- e. If you (use) a monitor with interlaced video for word processing, you (not use) a standard.
- f. If Godel’s incompleteness theorem(be not proved), rigorous and consistent philosophy of maths (be created) in the 20th century.

◆ Note 2

We can use *provided (that) / providing (that)* when we want to emphasize a condition.

It means if or only if.

Example: I would agree to these conditions provided (that) you increased my salary by 20%.

2.3 Make sentences from the following notes.

- a. experiment / would / have / give / more / reliable / results – it / have / be prepared / with / greater care.

- b. we / be able / start / this project / two / month – board / think / it / be / good idea.

 c. he / would / have / read / his paper – he / have / be given / time.

 d. I / will / send / you / fax - you / get / all information / you / need / today.

 e. whole thing / would not / have happened – they / have / be more careful.

 f. one / can / readily / find / length / third / side – one / know / length / two / side / triangle / and / measure / angle / between / they.

Read through the following situation. Say if you would do these things or not by putting a tick (✓) or a cross (✗) next to each of the sentences.

CULTURE QUIZ

- 1) If I were doing business in China and were asked about Taiwan, I would say “ It’s a country I have never visited”.
- 2) If I were having a meal with some Malay business colleagues in Kuala Lumpur, I would only pick up food with my right hand.
- 3) If I asked a Japanese businessman to do something and he said “Choto muzakashi” (It’s a little difficult), I would continue trying to persuade him to agree.
- 4) If I were invited to a British person’s home at 8 p.m for dinner, I would try and arrive 15 minutes late.
- 5) If I were doing business in Saudi Arabia, I would not speak Arabic unless I could speak it properly.
- 6) If I were in Oman, I would not start talking about business until after the second cup of coffee.

SKILLS DEVELOPMENT

● **Reading and speaking**

1. Pre – reading task

1.1 Use your dictionary to check the meaning of the following words.

- | | | |
|----------------|----------------------|----------------|
| mapping (n.) | visualize (v.) | emerge (v.) |
| essential (n.) | appropriately (adv.) | assertion (n.) |
| associate (v.) | | |

1.2 Fill each space with one of the given words.

- a. We only had time to pack a few
- b. One of the important concepts of mathematics is the notion of a
- c. I remember meeting him but I just can’t him.
- d. I seriously question a number of your

- e. He spoke to new project with his formal style.
- f. I wouldn't normally these two writers – their styles are completely different.

2. Read the text below.

MAPPINGS

Now, we shall concern ourselves with another of the important concepts of mathematics – the notion of a mapping. But first let us try an experiment designed to yield some information about our mental habits. Visualize your best friend. Of course, the image of a certain individual forms in your mind. But did you notice that accompanying this image is a name – the name of your friend? Not only did “see” your friend, but you also thought of his name. In fact, is it possible for you to visualize any individual without his name immediately emerging in your memory? Try ! Furthermore, is it possible for you to think of the name of an individual, at the same time, visualizing that individual? The point of the proceeding experiment is to demonstrate that we habitually link together a person and his name; we seldom think of one without the other. Let us see what there is of mathematical value in the above observation. First, let us state the essentials of the situation. On the one hand, we have a set of persons; on the other hand, the set of names of these persons.

With each member of the first set we associate, in a natural way a member of the second set. It is in the process of associating members of one set with the members of another set that something new has been created. Let us analyse the situation mathematically. Denote the set of persons by “P” and the set of names by “N”. We want to associate with each member of “ P ” an appropriately chosen member of N; in fact, we want to create a mathematical object which will characterize this association of members of N with members P. We rely on one simple observation: there is no better way of indicating that two objects are linked together than by actually writing down the names of the objects, one after the other; i.e., we indicate that two objects are associated by pairing the objects. Now we see the importance of ordered pairs. The ordered pair (a, b) can be used to indicate that a and b are linked together.

Now we know how to characterize associating members of N with members of P: construct the subset $P \times N$ obtained by pairing with each person his name. The resulting set of ordered pairs express mathematically the associating process described above, since the person and the name that belong together appear in the same ordered pair.

Now a definition. Let “A” and “B” denote any non empty set. A subset of $A \times B$, say μ , is said to be a mapping of A into B iff* each member of A is a first term of exactly one ordered pair in μ . Moreover, we shall say that the mapping μ associates with a given member of A, say a , the member of B paired with a . Thus, iff $(a, b) \in \mu$ we shall say that “b” is associated with “a” under the mapping μ , b is also called the image of a under the mapping. Note that the subset $P \times N$ constructed above is a mapping of P into N. Thus our notion of a mapping of A into N permits us to characterize mathematically the intuitive idea of associating a member of B with each member of A.

intuitive idea

mathematical representation



Under the intuitive idea, b is associated with a ; this is represented by the mathematical assertion $(a, b) \in \mu$. In short, the set μ characterizes the intuitive idea of associating a member of B with a member of A. If μ is a mapping of A into B such that each member of B is a second term of at least one member of μ , then we shall say that μ is a mapping of A into B. Furthermore, if a mapping of A into B such that no member of B is a second term of two ordered pairs in the mapping, then we shall say that this subset of $A \times B$ is one to one mapping of A into B. For example:

$\{ (1, 3), (2, 4), (3, 5), (4, 6) \}$ is one to one mapping of $\{1, 2, 3, 4\}$ into $\{1, 2, 3, 4, 5, 6, 7\}$. If μ is both a one to one mapping of A into B and a mapping of A onto B, then μ is said to be a one to one mapping of A onto B.

Comprehension check.

1. Answer the following questions.

- Which experiment in this text is designed to yield some information about our mental habit?
- Does the image of a man usually accompany his name?
- Does one necessarily visualize a man when hearing his name?
- Why do we habitually link together a person and his name?
- What do we show by pairing objects?
- How do we characterize associating members of N with the members of B?
- When do we say that μ is a mapping of A into B?
- Under what condition is a subset of $A \times B$ said to be a mapping of A into B ?
- What do we mean by saying that the subset of $A \times B$ is a one to one mapping of A into B ?

2. Discussion

* iff = if and only if

- Each mapping of A into B is also a mapping of B into A, isn't it?
- Given that B is a subset of C, show that each mapping of A into B is also a mapping of A into C.

3. Vocabulary

The following words are taken from the text. In each case, say whether the paired words are similar (S) or opposite (O).

a. associating – visualizing

b. intuitive – mental

- c. concept – idea
- d. demonstrate – analyse
- e. pair – accompany

● **Listening**

We are going to listen to some reasons to learn mathematics.

1. Work in pairs to discuss the questions.

- a. Do you think everyone needs to learn maths? Why or why not?
- b. Decide which of the following factors has made mathematics more necessary nowadays?
 - the growth of science and technology.
 - the advances in maths.
 - the wide spread use of electronic computers.
 - the specialized problems in maths.
 - the availability of electronic computers.
 - the use of computers.
 - the application of maths in business.

2. Listen to the tape. Check the factors mentioned.

TRANSLATION

Translate into Vietnamese.

1. Theorem 1:

Let t be a compact linear mapping of the separated convex space E into itself and let u and v be two commuting continuous linear mappings such that $u \circ v = v \circ u = \lambda i - t$ where $\lambda \neq 0$. Then E can be written as the topological direct sum of two (closed) vector subspaces M and N , each mapped into itself by v . On M , v is an isomorphism, while N is finite dimensional and on it v is nilpotent (i.e. there is an n with $v^n = 0$). For each positive integer r , $v^{-r}(0)$ and $E/v^r(E)$ have the same dimension. Finally, on E , v can be written in the form $v = v_1 + v_2$, where v_1 is an isomorphism of E onto itself and v_2 maps E into a finite-dimensional vector subspace.

2. Theorem 2: (Schauder.)

Let E and F be Banach spaces, E' and F' their duals with the norm topologies and t a weakly continuous linear mapping of E into F . Then t is compact if and only if t' is compact.

3. Theorem 3:

Suppose that E is a Banach space, with dual E' and bidual E'' , that F is a Banach space with dual F' , and that t is a weakly continuous linear mapping of E into F , with transpose t' and bitranspose t'' . Then the following are equivalent:

- * t maps bounded sets into $\sigma(F, F')$ – compact sets;
- * t' maps bounded sets into $\sigma''(E, E')$ – compact sets;
- * $t''(E'') \subseteq F$.

Just for fun

☺ THE BIG ZERO AND THE LITTLE ZERO

Once when a little zero fell on to a big zero, the big zero exclaimed “You stupid idiot ! Aren’t you ashamed of yourself ? Don’t you know how much bigger I am than you ?”.

The little zero replied: “What’s the point of your being bigger ? We are worth the same – nil ”.

UNIT 12

-ING / -ED RELATIVE PHRASES

PRESENTATION

1. Read the text below.

MULTIMEDIA

Welcome to the world of highend multimedia. Multimedia is not a new phenomenon, although it is new to business computing. We live in a multimedia world. At home, we experience a variety of media through our television: full motion video, still images, graphics, sound and animation. The situation described above is not quite here yet, but most of the pieces already exist to make this scenario become a reality using a network RS/6000 or other high-power workstation.

A manager creates a detailed business presentation involving text, graphics, digitized photographic still images and tables of spreadsheet data all combined in a single compound document. Before sending the document across the network to a colleague, the manager picks up the microphone and attaches an audio note to one of the tables, reminding the colleague about something unusual or potentially confusing in the accompanying figures.

2. Rewrite these sentences using relative clauses.

- a. The situation described above is not quite here yet.
- b. Most of the pieces already exist to make this scenario become a reality using a networked RS/6000 or other high powered workstation.

3. Grammar questions

- a. Which sentences express something is doing / was doing something at a particular time or permanent characteristics?
- b. Which sentences have passive meaning?
- c. Which words are left out in relative clauses?
- d. Which words do the present participle phrase modify?
- e. Which words do the past participle phrase modify?

4. Rewrite the sentences using an -ing / -ed phrases.

- a. A plane which (that) was carrying 28 passengers crashed into the sea yesterday.
.....
- b. None of the people who were invited to the party can come.
.....
- c. When I was walking home, there was a man who followed me.
.....

- d. All the letters that were posted yesterday should arrive tomorrow.
-

Rule:

There are two ways in which a relative clause is reduced to a participle phrase:

1. The and the form of the verb are omitted.
2. If there is no form of a verb in the relative clause, it is sometimes possible to omit the and change the verb to its –ing form.

PRACTICE

• **Grammar**

1. Change the relative clauses to participle phrases.

- a. Any expression like $x + 5$ or $2x - 3$ **that** contains two or more terms may be called a polynomial meaning an expression with many parts.
- b. An axiom is a statement **that** is generally accepted as true without proof.
- c. Most of the problems **that** concern digits (0 through 9) are based on the fundamental principle of our decimal system: that is, the position of a digit with respect to the decimal points indicates the value **which** is represented by it.
- d. Such quantities as 5 , x , $a - 1$ and $n^2 + 1$ are prime, since they are not divisible by any quantities **that** is except themselves and 1.
- e. The number $\frac{c}{d}$ or $\frac{c}{2r}$, **which** is the same for all circles, is designated by π .
- f. A diameter is a chord **which** passes through the centre of the circle.
- g. A circle is a set of points in a plane each of which is equidistant, that is the same distance from some given point in the plane **which** is called the centre.
- h. No matter how the problems which deal with the division of polynomials are stated they should always be copied in the form **that** is used for long division in arithmetic.
- i. Points A and B **that** represent the opposite points of a circle are equidistant from the centre.

◆ **Note**

Many verbs have irregular past participles which do not end in *-ed*.

For example: stolen, made, brought, written, ...

2. Change the participle phrases to relative clauses.

- a. The technician goes to a high power workstation attached to a network and calls up the information on the part and the replacement procedure.
- b. An image of the part seated in the engine appears.
- c. Instructions are first written in one of the high-level languages, e.g. FORTRAN, COBOL, ALGOL, PL/I, PASCAL, BASIC, or C, depending on the type of problem to be solved.

- d. Today the possibility of hackers breaking into corporate and government computers poses a constant threat to society.
- e. A program written in one of the languages is often called a source program and it can not be directly processed by the computer until it has been compiled.
- f. The compiler is a system program written in any language, but the computer's operating system is a true system program controlling the central processing unit (CPU), the input, the output and the secondary memory devices.
- g. Another system program is the linkage editor fetching required system routines and linking them to the object module.
- h. These features combined together provide a very powerful tool for the programmer.
- i. The program produced after the source program has been converted into machine code is referred to an object program or object module.

3. Change the following sentences according to the model.

- i**
 - A: – I have got a book which deals with computers.
 - B: – I've got a book dealing with computers.
 - a. I know the man who teaches you English.
 - b. Give me the journal which lies on the table.
 - c. I must see the scientists who work in this lab.
 - d. The letters which name the angles are A, B, C.

- ii**
 - A: – The material which is used in the article is true.
 - B: – The material used in the article is true.
 - a. The most prevalent calculator in the United States is the slide rule, which is based on the principle of logarithms.
 - b. One of the original calculators was undoubtedly a version of the Japanese abacus, which is still in use today.
 - c. Most calculators are based on the fundamental mathematical principle which is called the binary number system.
 - d. The calculators which were traced back to the Tigris Euphrates Valley 5000 years ago are original.

4. Complete these sentences using the verbs in the box.

stand	live	cry	steal	read	offer
knock	make	blow	call	wait	lead

- a. Somebody Jack phoned while you were out.
- b. When I entered the waiting room there was nobody except for a young man by the window a magazine.
- c. A few days after the interview, I received a letter me the job.
- d. Sometimes life must be very unpleasant for people near airports.
- e. The paintings from the museum haven't been found yet.
- f. Did you hear about the boy down on his way to school this morning?
- g. Most of the suggestions at the meeting were not very practical.

- h. At the end of the street there is a path to the river.
- i. I was woken up by the baby
- j. There was a tree down in the storm last night.

SKILLS DEVELOPMENT

• Reading

1. Pre – reading task

Work in pairs. Choose the best definition of matrix.

- a. An array of numbers arranged in rows and columns.
- b. A set of quantities (called elements) arranged in a rectangular array, with certain rules governing their combination.
- c. An n–dimensional real vector space containing a set of real numbers.

2. Read the text below.

MATRICES

Although the idea of a matrix was implicit in the quaternion (4–tuples) of N.Hamilton and also in the “extended magnitude” (n–tuples) of H.Grassmann, the credit for inventing matrices is usually given to Cayley with a date of 1857, even though Hamilton obtained one of two isolated results in 1852. Cayley said that he got the idea of a matrix “ either directly from that of a determinant, or as a convenient mode of expression of the equations $x' = ax + by$, $y' = cx + dy$ ”. He represented this transformation and developed an algebra of matrices by observing properties of transformations on linear equations:

$$\begin{cases} x' = ax + by \\ y' = cx + dy \end{cases} \rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Cayley also showed that a quaternion could be represented in matrix form as shown above where a, b, c, d are suitable complex numbers. For example, if we let the quaternion units 1, i, j, k be represented by

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$$

the quaternion $4 + 5i + 6j + 7k$ can be written as shown below:

$$\begin{bmatrix} 4 + 5i & 6 + 7i \\ -6 + 7i & 4 - 5i \end{bmatrix}$$

This led P.G.Tait, a disciple of Hamilton, to conclude erroneously that Cayley had used quaternion as his motivation for matrices. It was shown by Hamilton in his theory of quaternion that one could have a logical system in which the multiplication is not commutative. This result was undoubtedly of great help to Cayley in working out his matrix calculus because matrix multiplication also is non–commutative. In 1925 Heisenberg discovered that the algebra of matrices is just right for the non–commutative maths describing phenomena in quantum mechanics.

Cayley's theory of matrices grew out of his interest in linear transformations and algebraic invariants, an interest he shared with J.J.Silvester. In collaboration with J.J.Silvester, Cayley began the work on the theory of algebraic invariants which had been in the air for some time and which, like matrices, received some of its motivation from determinants. They investigated algebraic expressions that remained invariants (unchanged except, possibly, for a constant factor) when the variables were transformed by substitutions representing translations, rotations, dilatations ("stretching" from the origin), reflections about an axis, and so forth.

There are three fundamental operations in matrix algebra: addition, multiplication and transposition, the last not occurring in ordinary algebra. The law of multiplication of matrices which Cayley invented and his successors have approved, takes its rise in the theory of linear transformations. Linear combinations of matrices with scalar coefficients obey the rules of ordinary algebra. A transposition is a permutation which interchanges two numbers and leaves the other fixed, or in other words: the formal operation leading from x to x' and also that leading from x' to x is called transposition. A matrix of m rows and n columns has rank r , when not all its minor determinants of order r vanish, while of order $r + 1$ do. A matrix and its transposition have the same rank. The rank of a square matrix is the greatest number of its rows or columns which are linearly independent.

Today, matrix theory is usually considered as the main subject of linear algebra, and it is a mathematical tool of the social scientist, geneticist, statistician, engineer and physical scientist.

Comprehension check

1. Are these statements true (T) or false (F)? Correct the false statements.

- a. Cayley was the first inventor of matrices.
- b. Cayley's idea of a matrix comes from Hamilton's theory of quaternion.
- c. Properties of transformations on linear equations serve as the basis of Cayley's theory of matrices.
- d. The law of multiplication of matrices does not relate to the theory of linear transformations.

2. Answer the following questions.

- a. Who was the first to create a matrix? When was this?
- b. Did anyone obtain the idea of a matrix before him?
- c. What did Hamilton show in his theory of quaternion?
- d. Why did Hamilton's theory of quaternion help Cayley to work his out matrix theory?
- e. What did Heisenberg discover in 1925?
- f. Who did Cayley collaborate with on the theory of algebraic invariants? What did they investigate?
- g. What are three fundamental operations in matrix algebra?

3. Vocabulary

The Second irregular plural forms of nouns of Latin and Greek origin.

◆ **Note:**

A number of Latin and Greek original nouns have plural forms ending in –es, for example: matrix → matrices.

3.1 Write the plural forms of these nouns:

– analysis	→.....	– hypothesis	→.....
– axis	→.....	– index	→.....
– basis	→.....	– parenthesis	→.....
– crisis	→.....	– synthesis	→.....
– directrix	→.....	– thesis	→.....
– emphasis	→.....	– vertex	→.....

Complete the rule:

– The ending –....., –..... / –..... are changed into –..... in the plural forms.

3.2 Fill in the gaps using the words above.

- Points A, B, C are called the of triangle ABC.
- In $b^{3n} + x$, 3 and x are
- Rates of work are calculated on a monthly
- The vertical line is named the y
- Nowadays, some schools put great..... on foreign language study.
- It's always not easy to prove a

• Listening

Listen to the passage about William Rowan Hamilton.

1. Pre– listening task

a. Use your dictionary to check the words below.

- | | |
|--------------------|------------------------|
| – academy (n) | – observatory (n) |
| – appoint (v) | – quintic equation (n) |
| – contribution (n) | – scratch (v) |

- flash (v)
- three–dimensional complex number (n)

b. Complete sentences using the words above.

- Hamilton presented his Theory of system of Rays to the Royal Irish
- He was astronomer royal at Dunsink and professor of astronomy at Trinity College.
- Hamilton was interested in which he called “triplets”.
- Hamilton’s major were in algebra of quaternion, optics and dynamics.
- The discovery of quaternion in his mind while he was walking along The Royal Canal.
- He the formula on the stones of a bridge over the canal.

2. Listen to the tape. Answer the following questions.

- a. When and where was William Rowan Hamilton born?
- b. What did he do on April 23, 1827?
- c. How old was he when he was appointed astronomer at Dunsink Observatory and professor of astronomy at Trinity College?
- d. To which scientific field did he mainly contribute?
- e. What did he call three–dimensional complex numbers?
- f. What happened to him on October 16, 1843?
- g. What did he do in 1837?

TRANSLATION

• **Translate into Vietnamese.**

L’Hospital’s rule:

A rule for finding the limit of the ratio of two functions each of which separately tends to zero. It states that for two functions $f(x)$ and $g(x)$, the limit of the ratio $\frac{f(x)}{g(x)}$ as $x \rightarrow a$ is equal to the limit of the ratio of the derivatives $\frac{f'(x)}{g'(x)}$ as $x \rightarrow a$.

For example, the functions $x^2 - 4$ and $2x - 4$ have a ratio $\frac{x^2 - 4}{2x - 4}$. As $x = 2$, this ratio takes the indeterminate form $\frac{0}{0}$; i.e. the limit of ratio cannot be found directly. L’Hospital’s rule states that the limit of the ratio is equal to the limit of the ratio of the first derivatives; i.e. the limit of $\frac{2x}{2}$ as $x \rightarrow 2$, is 2. If the ratio of first derivatives is also indeterminate, higher order derivatives can be used.

• **Translate into English.**

1. Ma trận cấp $m \times n$ là tập hợp của mn số được sắp đặt trong một bảng chữ nhật với m dòng và n cột.
2. Phép cộng ma trận:

Phép cộng hai ma trận chỉ được thực hiện trên hai ma trận có cùng số dòng và cùng số cột. Bằng cách cộng các phần tử tương ứng để cấu tạo nên một ma trận có cùng số dòng và cùng số cột ta được tổng của chúng.

Ví dụ, nếu $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 0 & 9 \end{bmatrix}$ và $B = \begin{bmatrix} -2 & 4 \\ 0 & 7 \\ -4 & 5 \end{bmatrix}$ thì:

$$A + B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 0 & 7 \\ -4 & 5 \end{bmatrix} = \begin{bmatrix} 1+(-2) & 2+4 \\ 3+0 & 4+7 \\ 0+(-4) & 9+5 \end{bmatrix} = \begin{bmatrix} -1 & 6 \\ 3 & 11 \\ -4 & 14 \end{bmatrix}$$

3. Phép chuyển vị ma trận:

Ma trận chuyển vị A^T của ma trận A là một ma trận được thành lập bằng cách đổi chỗ các dòng của A làm cột tương ứng với ma trận mới. Ví dụ, nếu: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

thì: $A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$, trong đó dòng thứ nhất và dòng thứ hai trở thành cột thứ nhất và thứ hai của A^T .

Just for fun ☺ A COUNTING PROBLEM

- Two fathers and two sons, how many people are there in the total?
- Too easy! Four!
- Wrong!
- Why is it wrong?
- I am the son of my father. My father is the son of my grand father. So two fathers and two sons are three people. Is that correct?

UNIT 13

PAST PERFECT SIMPLE & PAST PERFECT CONTINUOUS

PRESENTATION

1. *Read the text below.*

MATHEMATICS AND MODERN CIVILIZATION

“Mathematics is the queen of natural knowledge”. (K.F.Gauss)
That is true. Maths supplies a language, methods and conclusions for science. It enables scientists to predict result, furnishes science with ideas to describe phenomena and prepares the minds of scientists for new ways of thinking.

It would be quite wrong to think that maths had been giving so much to the sciences and receiving nothing in return. Physical objects and observed facts had often served as a source of the elements and postulates of maths. Actually, the fundamental concepts of many branches of maths are ones that had been suggested by physical experiences.

Scientific theories have frequently suggested directions for pursuing maths investigations, thus furnishing a starting point for maths discoveries. For example, Copernican astronomy had suggested many new problems involving the effects of gravitational attraction between heavenly bodies in motion. These problems had been developing the further activities of many scientists in the field of differential equations.

2. Grammar questions

2.1 *Answer the following questions.*

- a. What had Copernican astronomy suggested?
- b. Which problems had been developing the further activities of many scientists in the field of differential equations?

2.2 *Work in pairs to compare the use of tenses in the following answer pairs. Say which tenses are used?*

- a. Copernican astronomy had suggested many new problems.
- b. The problems involving the effects of gravitational attraction between heavenly bodies in motion had been developing.
 - Which sentence expresses an activity that was completed before another activity or time in the past?
 - Which sentence emphasizes the duration of an activity that was in progress before another activity or time in the past?

Find another examples used the past perfect simple and the past perfect continuous from the text.

Complete the rules:

The past perfect simple is formed with + the

The past perfect continuous is formed with + + the

PRACTICE

1. Grammar

1.1 Underline the correct verb form: Past perfect simple or past perfect continuous.

- Ever since Galileo (had invented / had been inventing) his telescope men (had studied / had been studying) the motions of the planets with ever increasing interest and accuracy.
- Kepler (had deduced / had been deducing) his famous three laws describing the motion of the planets about the sun.
- The Englishman Thomas Harriot was the first mathematician who (had given / had been giving) status to negative numbers.
- We knew the solution of this problem because we (had read / had been reading) it in maths magazine.
- Amalie Emmy Noether (had published / had been publishing) a series of papers focusing on the general theory of ideas for four years from 1922 to 1926.
- When W. Hamilton (had walked / had been walking) along the Royal canal all day, he (had discovered / had been discovering) the multiplication formula that has been used for the quaternions on the stones of a bridge over the canal.

1.2 Put in the correct form of the verbs.

- Emma went into the sitting room. It was empty, but the television was still on. Some one (watch) it.
- I (play) tennis, so I had a shower. I was annoyed because I (not win) a single game.
- The walkers finally arrived at their destination. They (walk) all day, and they certainly needed a rest. They (walk) thirty miles.
- I found the calculator. I (look for) it for ages.
- I finally bought a new calculator. I (look) everywhere for the old one.

2. Reading and speaking

2.1 Read each situation and then tick the right answer.

Example: Two men delivered the sofa. I had already paid for it.
Which came first, a) the delivery, or b) the payment?

- a. The waiter brought our drinks. We'd already had our soup.
Which came first, a) the drinks, or b) the soup?
- b. I'd seen the film, so I read the book.
Did I first, a) see the film, or b) read the book?
- c. The programme had ended, so I rewound the cassette.
Did I rewind the cassette, a) after, or b) before the programme ended?
- d. I had an invitation to the party, but I'd arranged a trip to London.
Which came first, a) the invitation, or b) the arrangements for the trip?

2.2 Continue the sentences. Say what activities had been going on.

Example: He felt very tired at 4.30 because he had been working at the VDU* all day.

- a. They realized that none of their confidential information was safe because
- b. She felt that a change of job would be good for her because
- c. The accountant finally discovered why the phone bill was so high.
One of the night security guards
- d. There was a very long delay at the airport. When we finally left, we

3. Vocabulary

Adverbs.

3.1 Many adverbs in English are formed by adding – ly to the adjective. These adverbs have the same meaning as the adjectives. For example: beautiful – beautifully, careful – carefully, ...

Example: He spoke English badly = His English was bad.

Form adverbs from following adjectives:

close –.....	most –.....
fair –.....	mere –.....
high –.....	large –.....
ready –.....	short –.....

3.2 But there are also many adverbs that don't end in –ly: even, still, too, then, only, always, together, ...

Example: He still lives in England.

3.3 Put the adverbs from the box into one of the spaces.

readily	lately	largely	shortly	thus
too	mostly	then	still	

even	fairly	hence	hardly
------	--------	-------	--------

* VDU: visual display unit

- a. This concept was narrow and a better understanding of a function was needed.
- b. The Arabs acquired most the Greek and Hindu scientific writings which they translated into Arabic and preserved through the Dark Ages of Europe.
- c. Descartes' "La Geometrie" consists of what we now call the "theory of equations" and it contains his famous rule of signs for determining the number of positive and negative roots of an equation.
- d. However, when we use the formal definitions, we distinguish between the function and the graph.
- e. ... By the theorem we know that f^{-1} is a function....., the composition of f by f^{-1} is also a function.
- f. More than two hundred algebraic structures have been studied
- g. after Galois was killed in a duel in 1832, the development of group theory was substantially advanced by Cauchy.
- h. In the symbolic stage algebraic notation went through many modifications and changes until it became stable by the time of Newton.
- i. A great many persons have been involved in developing the foundations of modern algebra, the members of the British School of Algebraists.
- j. The contributions of Abel and Galois to modern algebra can be overestimated.
- k. To solve this problem, we observe that if the length of the sandbox is a ft,the width is $\frac{25}{a}$, the amount of wood required is $25+2a+\frac{50}{a}$ sq. ft .

SKILLS DEVELOPMENT

• **Reading and speaking**

1. Here are some words that may cause problems for you.

Use your dictionary to help.

- | | |
|------------------|---------------|
| - function (n) | - tangent (n) |
| - domain (n) | - graph (n) |
| - derivative (n) | - plot (v) |
| - slope (n) | - sketch (v) |

2. *Find the sentences that contain these words in the text. Translate them into Vietnamese.*

THE DERIVATIVE OF A FUNCTION AND SOME APPLICATIONS OF THE DERIVATIVE

Given a function, say f , we want to associate a number with any given number in the domain of f ; let us agree to denote this set by writing Df . In effect, this means that we want to construct a function Df from the given function f . The function that we shall construct from f is called the derivative of f and is denoted by writing f' .

The concept of the derivative of a function has a number of important applications. One of these applications is concerned with the problem of sketching the graph of a function. Of course, since the graph of the function is the set of points of the plane that correspond to the members of the function, we see that there is no theoretical difficulty involved in this problem – we merely locate the points of the plane corresponding to the ordered pairs of the function. The fact that there usually is an infinite number of ordered pairs in the function being graphed means that the few points actually plotted must be chosen with some care, so that a representative picture of the function is obtained. Even this can be avoided by the simple device of plotting

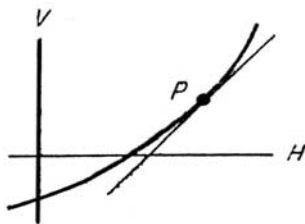


Fig. 1

many points. But the mathematician had been extremely lazy and rather than carry out the time taking monotonous job of plotting many points; he had been sitting and thinking a moment with the hope of finding a way of avoiding such boredom.

One may define the function f' in such a manner that for any number a , $f'(a)$ is the slope of the tangent to the graph of f at the point $(a, f(a))$ on the graph.

We will proceed as follows. The tangent of a curve at a point, P , on the curve is an important concept largely because a small segment of the tangent line containing P differs very little from the curve (see Figure 1). This suggests to the lazy mathematician that when he plots a point on the graph he should determine the slope of the tangent line to the curve at that point (this is easily accomplished by consulting the derivative) and then draw in a short segment of the tangent at that point. In this way a number of line segments is obtained, rather than points alone. For instance, let us obtain the graph of the function $x^2 + 2x - 3$. The ordered pairs $(-4, 5)$, $(-3, 0)$, $(-2, -3)$, $(-1, -4)$, $(0, -3)$, $(1, 0)$, $(2, 5)$ are each members of the function, and the corresponding points are easily plotted. Joining these points by a smooth curve (dotted in the diagram), we had obtained a sketch of the graph (see Figure 2). Let us now make use of the derivative. The derivative of our function is $2x + 2$, and so the tangent lines at each of the seven points plotted are: $-6, -4, -2, 0, 2, 4, 6$. Short line segments having these slopes have been drawn through the corresponding points of the diagram.

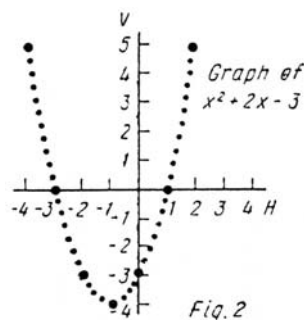


Fig. 2

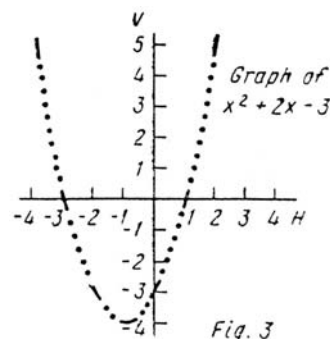


Fig. 3

Finally, the line segments are joined by a smooth curve (dotted in the diagram), and so the graph of $x^2 + 2x - 3$ has been sketched (see Figure 3). Examining Figure 2 we do not feel quite sure about the resulting graph, since so few points have been plotted, but in Figure 3 the line segments plotted seem to be joined up by a smooth curve.

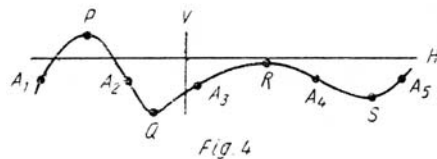
Comprehension check

1. Answer the following questions.

- What is the derivative of a function? How is it denoted?
- Is the problem of sketching a graph of a function concerned with the concept of the derivative?
- Is there a difficulty involved in sketching a graph? Why or why not?
- Why must the few points actually plotted on the graph be chosen with some care?
- In what way can the lazy mathematician define a function f' ?
- In what way can a number of line segments be obtained?
- How had we obtained a sketch of the graph?
- Can we feel sure about the resulting graph? Why or why not?

2. Discussion

Look at figure 4 and say what the easiest way of sketching the graph is?



• Listening

- Here are several styles of functions which have special names in the mathematical world. *Try to understand them.*

one-to-one function	linear function
rational function	constant function
identity function	inverse function

- Listen to the tape. Find a definition for each style of function. Complete the definitions.*

- If A is a non-empty set, a function with domain A and with range a set with only one element, is called a
- If a and b are real numbers, a function g defined by $g : x \rightarrow ax + b$ is called a

- c. If A is any non-empty set, the function I_A defined by $I_A(x) = x$ for each $x \in A$ is called on A .
- d. Let f and g be two real (or complex) polynomial functions. Then the function h defined by $h(x) = f(x)/g(x)$ is called a
- e. A function f is said to be if, and only if, no two distinct members of f have the same second element.
- f. If f is a function such that the inverse relation f^{-1} is also a function, then f is said to have

TRANSLATION

1. Translate the biography of a mathematician into Vietnamese.

He was born on August 5, 1802, in Finnøy, Norway. He was the second of six children. At the age of 13 he was sent to the Cathedral school in Christiania (Oslo).

In 1817 his maths teacher Holmbe recognized his talent and started giving him special problem and recommended special books outside the curriculum. Soon he became familiar with most of the important mathematical literature.

When he was 18, his father died. He had to support his family. He gave private lessons and did odd jobs. However, he continued to carry out his mathematical research.

In his last year of school, he worked on the problem of the solvability of the quintic equation, a problem that had remained since the sixteenth century. Unable to find an error and unable to understand his arguments, he was asked by the editor to illustrate his method. In 1824, during the process of illustration he discovered an error. This discovery led him to a proof that no such solution exists. He also worked on elliptic functions and in essence revolutionized the theory of elliptic functions.

He traveled to Paris and Berlin to find a teaching position. Then poverty took its toll and he died from tuberculosis on April 6, 1829. Two days later a letter from Crelle reached his address, conveying the news of his appointment to the professorship of mathematics at the University of Berlin.

2. Can you guess this mathematician's name?

Was he: – Emst Steinitz,
 – Niels Henrik Abel,
 or Joseph Louis La Grange?

Additional text

1. Guess the meaning of these words.

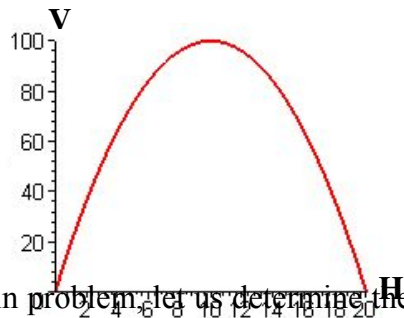
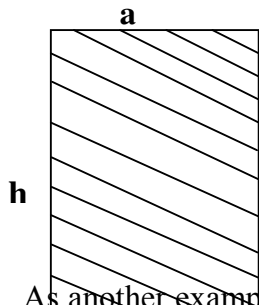
variety, critical, hold, minimum, maximum, memorize, maximize, summarize.

2. Read the text.

MAX – MIN PROBLEMS

Let's consider the problem of determining the dimensions of a rectangle that has the largest possible area, subject to restriction that the perimeter of the rectangle is 40 ft.

Suppose that the dimensions of rectangle are a and h (see figure 5). Then the perimeter is $2.(a + h)$ and therefore $h = 20 - a$. But the area of the rectangle is $a.h = a.(20 - a)$. This leads us to consider the function A where $A = \{ (a, b) / b = a.(20 - a) \text{ and } 0 < a < 20 \}$. Note that the second term of an ordered pair in A is the area of the rectangle one of whose dimensions is the first term of that ordered pair: All possible rectangles, subject to the restriction of the problem have been captured, in this sense, by the function A . We want to determine the ordered pair in A with the largest second term. This is accomplished by sketching the graph of the function $f = 20x - x^2$ (see figure 6). Clearly, $f' = 20 - 2x$ and $f'' = -2$. By examining the first derivative we see that function is critical at 10, and by examining, the second derivative we see that the function has a relative maximum at 10. Note that $f(5) = 75$, $f'(5) = 10$, $f(15) = 75$, $f'(15) = -10$, $f(10) = 100$, $f'(10) = 0$, $f(20) = 0$, $f'(20) = -20$. Using this information, we obtain the sketch of $20x - x^2$ shown in the diagram. Now we can see that the function A has its maximum value at the number at which A is a relative maximum, namely at 10, the desired rectangle therefore having dimensions 10 ft \times 10 ft. In general, the maximum value of a function is the largest of the values the function takes at the numbers at which the function is a relative maximum; but care must be taken to ensure that this value is not exceeded by the value of the function at a boundary of the domain of the function.



As another example of a max-min problem, let us determine the length and the width of a sandbox whose height is 1 ft and which hold 25 cub ft of sand, in other that the minimum amount of wood will be used in constructing it.

To solve this problem, we observed that if the length of the sandbox is a ft, then the width is $\frac{25}{a}$; hence the amount of wood required is $25 + 2a + \frac{50}{a}$ sq. ft. Thus we are concerned with the function W , where

$$W = \{(a, b) | b = 25 + 2a + \frac{50}{a} \text{ and } 0 < a\}$$

Note that the second term of an ordered pair in W represents the amount of wood required to construct the sandbox whose

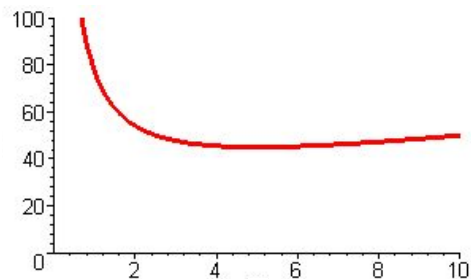


Fig.7

length is given by the first term of the ordered pair. Therefore we want to find the number at which the function W takes its minimum value. We accomplish this by sketching the function $25 + 2x + \frac{50}{x}$ (see figure 7). The first derivative of this function is $2 - \frac{50}{x^2}$ and the second derivative is $\frac{100}{x^3}$. The function is critical at 5 and at -5 . We conclude that the function is a relative minimum at 5 and a relative maximum at -5 . However, we are interested only in the part of the graph to the right of V: $W(5) = 45, W(1) = 77$ and $W'(1) = -48, W(10) = 50$ and $W'(10) = \frac{3}{2}$.

Using this information, we obtain the sketch of W shown in the diagram. Thus the minimum value of W is at 5, the relative minimum of W . And so the length of the sandbox is 5 f t. Summarizing, in order to solve a max–min problem, we first determine the function which we wish to maximize or minimize and then calculate the first and the second derivatives of this function. The zeros of the first derivative include all relative maxima and relative minima of the function, whereas the second derivative distinguishes between a number at which a function is a relative maximum and one at which the function is a relative minimum. Finally as a check, we sketch the function.

3. Answer the following questions.

- What kind of problems are dealt with in this text?
- See fig.5 and say how you determine the perimeter of the rectangle?
- What is the area of the rectangle equal to?
- What is the maximum value of the function?
- Is it possible for the maximum value of a function to be exceeded by the value of the function at a boundary of the domain of the function?
- What is the minimum value of the function?
- What are we to do first if we wish to solve a max–min problem?
- What do the zeros of the first derivation include?

Just for fun

☺ SUBTRACTION

The teacher said: “When we subtract, we must have things of the same kind. We can’t take three kittens from four puppies, or a pound of meat from ten heads of oxen, or four hens from twelve bears, or seven nuts from eleven apples. Is that clear? Do you understand?”

“Yes, teacher.” Answered a boy, “but why can we take five roses from eight bushes?”

UNIT 14

REPORTED STATEMENTS

PRESENTATION

A market research organization interviewed a number of people to answer the questions: “Do you think the use of virtual reality in computer war games is going to affect young people’s attitude to violence?”

Here are some responses.



Rita Harper

‘Yes, I do. I think anything which portrays violence as fun is going to alter young people’s perception of violence in a very dangerous way. Violent crime amongst young people is increasing. I think manufactures of computer war games must take some of the responsibility.’



Susan Clark

‘No, not really, Kids – particularly boys – have been playing with toy guns ever since guns were invented. Surely playing with toy guns in the real world is more dangerous than playing with imaginary guns in an imaginary world.’



Mark Watts

‘It’s difficult to say. Some of my friends get very aggressive when they play computer war games. But I don’t really know if it makes them more violent when they’re doing other things. I play a VR jet fighter game, and I don’t think it has made me more violent.’

1. What did Rita, Susan and Mark actually say?

- Rita:
- I think anything which portrays violence as fun is going to alter young people’s perception of violence in a very dangerous way.
 - Violent crime amongst young people is increasing.
 - I think manufactures of computer war games must take some of the responsibility.

Do the same for Susan and Mark’s opinions.

2.1 Suppose that you’re working for a market research organization. *Write the article after researching. Let’s begin:*

Rita Harper said that she thought anything which portrayed as fun was going to alter young people's perception of violence in a very dangerous way. Violent crime amongst young people was increasing. Finally, she told us that manufactures of computer war games had to take some of the responsibility.

2.2 Continue your writing with Susan and Mark using some verbs such as: announce, answer, explain, mention, promise, reply, say, suggest, tell, ...

• **Grammar questions**

- a. In what tense is the main verb in reported speech?
- b. How are the verb tenses in reported statement different from those in the original words?

Complete the table:

<i>Direct speech</i>	<i>Indirect speech</i>
Simple Present Past Continuous
Present Perfect
Simple Past Would

PRACTICE

1. Grammar

◆ **Note 1**

Changing direct speech to reported speech:

We may have to make changes when we are reporting something which another person has said, or when we report it in a different place or at a different time. Here are some typical changes:

- Person:** I → he / she
my → his / her
- Place:** here → there / at the flat ...
- Time:** now → then / at that time
today → that day, on Monday, etc
yesterday → the day before / the previous day
tomorrow → the next / following day, on Sunday, etc
this week → that week
last week → the week before, the previous week
an hour ago → an hour before

1.1 Turn direct speech into indirect (reported) speech.

- a. Plato advised, “The principal men of our state must go and learn arithmetic, not as amateurs, but they must carry on the study until they see the nature of numbers with the mind only.”
- b. Descartes, father of modernism, said, “All nature is a vast geometrical system. Thus all the phenomena of nature are explained and some demonstration of them can be given.”
- c. In Descartes’ words, “You give me extension and motion then I’ll construct the universe.”
- d. The often repeated motto on the entrance to Plato’s Academy said, “None ignorant of geometry enter here.”
- e. J.Kepler affirmed: “ The reality of the world consists of its maths relations. Maths laws are true cause of phenomena. ”
- f. I.Newton said, “ I don’t know what I may appear to the world; but to myself I seem to have been only like a boy playing on the seashore, and diverting myself now and then by finding a smoother pebble or a prettier shell than usual; whist the great ocean of truth lay all undiscovered before me. If I saw a little farther than others, it is because I stood on the shoulders of giants ”.

◆ **Note 2**

If the statement is still up to date when we report it, then we have a choice. We can either leave the tense the same, or we can change it.

Example: Sarah said she’s going / was going to Rome in April.

1.2 Who said what? Match the words to the people and report what they said.

- | | | |
|------------------------|----|--------------------------------------|
| a. Mrs. Thatcher | 1. | “All the world’s a stage.” |
| b. Stokeley Carmichael | 2. | “Black is beautiful.” |
| c. Galileo | 3. | “Big Brother is watching you.” |
| d. Shakespeare | 4. | “There is no such thing as society.” |
| e. George Orwell | 5. | “The earth moves round the sun.” |

- a.
- b.
- c.
- d.
- e.

1.3 Circle the best answer.

Example: What did that man say3.....?

- 1) at you 2) for you ③ to you 4) you

a. I rang my friend in Australia yesterday, and she said it ... raining there.

- 1) is 2) should be 3) to be 4) was

- b. The last time I saw Jonathan, he looked very relaxed. He explained that he'd been on holiday the week.
 1) earlier 2) following 3) next 4) previous
- c. Someone me there's been an accident on the motorway.
 1) asked 2) said 3) spoke 4) told
- d. When I rang Tessa some time last week, she said she was busyday.
 1) that 2) the 3) then 4) this
- e. Judy going for a walk, but no one else wanted to.
 1) admitted 2) offered 3) promised 4) suggested
- f. When he was at Oliver's flat yesterday, Martin asked if he use the phone.
 1) can 2) could 3) may 4) must

2. Speaking

Work in pairs. Read the report about what a candidate said at an interview. Change the words in italic into direct speech.

Miss Lan said that *she was very interested in teaching there*, and she explained that *she had been working in a department of maths for three years*. When I asked her about her reasons for leaving, she said that *she liked young people and she wanted more responsibility*. She seems well qualified in the computer, as she said that *she had a degree in Informatics–Maths*. As far as her terms of notice are concerned, she made it clear that *she couldn't leave her job for another month*. I decided to offer her the job, and she said *she would consider our offer, and would let us have her decision soon*.

Example: A: – She said she was very interested in teaching there.
 B: – “I'm very interested in teaching here.”

3. Vocabulary

The Third irregular plural nouns of Latin and Greek origin (continued)

Except for two cases we have known, a lot of Latin and Greek original nouns also have the plural forms ending in –i (e.g. calculus – calculi) and –ae (e.g. abscissa – abscissae).

Look at the nouns below:

focus, formula, corona, genius, locus, hyperbola, lacuna, radius, nebula, modulus, nucleus, rhombus.

3.1 Put them in the correct column, writing in their plural forms;

–us → –i	–a → –ae
focus – foci	hyperbola – hyperbolae
.....
.....
.....
.....

3.2 *Fill in the gaps using the words above.*

- Two equations are called equivalent if they have the same
- Up until quite recently, when functions were mentioned in the mathematical literature they were usually considered to be
- In the figure, we can sketch the determined by an equation of the form.
- The simplest, that of common hydrogen, has a single proton.
- The area of an ellipse equals $\pi/4$ times the product of the long and the short diametres or π times the product of the long and the short

◆ Note

Besides that, some nouns always have similar forms, example:

- an apparatus – apparatus
- a headquarters – headquarters
- a means – means
- news – news
- a series – series
- a species – species

SKILLS DEVELOPMENT

● Reading

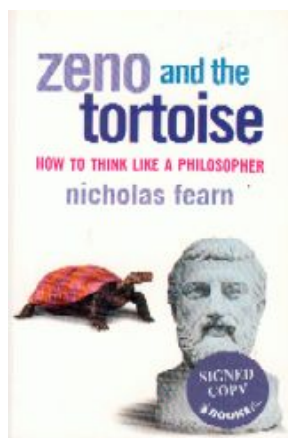
1. Pre – reading task

Let us solve the problem.

Achilles and a tortoise run a race in which the slow tortoise is allowed to start from a position that is ahead of Achilles' starting point. It is agreed that the race is to end when Achilles overtakes the tortoise. At each instant during the race Achilles and the tortoise are at some point of their paths and neither is twice at the same point. Then, since they run for the same number of points, the tortoise runs through as many distinct points as does Achilles. Can Achilles overtake the tortoise?

2. Read the text below to find the answer.

ZENO'S PARADOXES



There are difficulties in maths concepts of length and time which were first pointed out by the Greek philosopher Zeno, but which can now be resolved by use of Cantor's theory of infinite classes. We've just considered a formulation by Bertrand Russell of Zeno's Achilles and the tortoise paradox.

Part of this argument is sound. We must agree that from the start of the race to the end the tortoise passes through as many points as Achilles does, because at each instant of time during which they run each occupies exactly one position.

Hence, there is a one-to-one correspondence between the infinite set of points run through by the tortoise and the infinite set of points run through by Achilles. The assertion that because he must travel a greater distance to win the race Achilles will have to pass through more points than the tortoise is not correct, however, because as we know the number of points on the line segment Achilles must traverse to win the race is the same as the number of points on the line segment the tortoise traverses. We must notice that the number of points on a line segment has nothing to do with its length. It is Cantor's theory of infinite classes that solves the problems and saves our math theory of space and time.

For centuries mathematicians misunderstood the paradox. They thought it merely showed its poser Zeno was ignorant that infinite series may have a finite sum. To suppose that Zeno did not recognize it is absurd. The point of the paradox could not be appreciated until maths passed through the third crisis. Cantor holds that it does make sense to talk of testing an infinity of cases. The paradox is not that Achilles doesn't catch the tortoise, but that he does.

In his fight against the infinite divisibility of space and time Zeno proposed other paradoxes that can be answered satisfactorily only in terms of the modern math conceptions of space and time and the theory of infinite classes. Consider an arrow in its flight. At any instant it is in a definite position. At the very next instant, says Zeno, it is in another position. There is no next instant, whereas the argument assumes that there is. Instants follow each other as do numbers of the number system, and just as there is not next larger number after 2 and $2\frac{1}{2}$, there is no next instant after a given one. Between any two instants an infinite number of others intervene.

But this explanation merely exchanges one difficulty for another. Before an arrow can get from one position to any nearby position, it must pass through an infinite number of intermediate positions, one position corresponding to each of the infinite intermediate instants. To traverse one unit of length an object must pass through an infinite number of positions but the time required to do this may be no more than one second; for even one second contains an infinite number of instants. There is, however, a greater difficulty about motion of the arrow. At each instant of its flight the tip of the arrow occupies a definite position. At that instant the arrow cannot move, for an instant has no duration. Hence, at each instant the arrow is at rest. Since this is true at each instant, the moving arrow is always at rest. This paradox is almost startling; it appears to defy logic itself. The modern theory of infinite sets makes possible an equally startling solution. Motion is a series of rests. Motion is nothing more than a correspondence between positions and instants of time, the positions and the instants each forming an infinite set. At each instant of the interval during which an object is in "motion" it occupies a definite position and may be said to be at rest.

The maths theory of motion should be more satisfying to our intuition for it allows for an infinite number of "rests" in any interval of time. Since this concept of motion also resolves paradoxes, it should be thoroughly acceptable. The basic concept in the study of infinite quantities is that of a collection, a class, or a set of instants in

time. Unfortunately, this seeming simple and fundamental concept is beset with difficulties, that revealed themselves in Zeno's paradoxes.

Comprehension check

1. Answer the following questions.

- a. Why are the points passed through by Achilles and the tortoise equal?
- b. Did mathematicians at that time agree with Zeno's paradox? What did they think about it?
- c. Does it make sense to speak of completing an infinite series of operations in Cantor's opinion?
- d. What did Zeno propose to fight against the infinitive divisibility of space and time?
- e. Why is not Zeno's explanation satisfactory?
- f. Does the maths concept of motion satisfy the conception of physical phenomenon of motion? Why or why not?
- g. What is the basic concept in the study of infinite numbers?

2. Find words or phrases from the text that fixed the meaning of the underlined words.

- a. At any moment the arrow is in another position.
- b. The amount of points on a line segment is not concerned with its length.
- c. Between any two instants there is an infinite number of other intermediate ones.
- d. The number of points that Achilles and the tortoise passed is equal.
- e. At each instant of the period of time during which an object is in motion, it occupies a definite position.
- f. Lots of people thought that the paradox is unreasonable.

• Writing

Put the sentences in a suitable order to have a short biography of George Cantor.

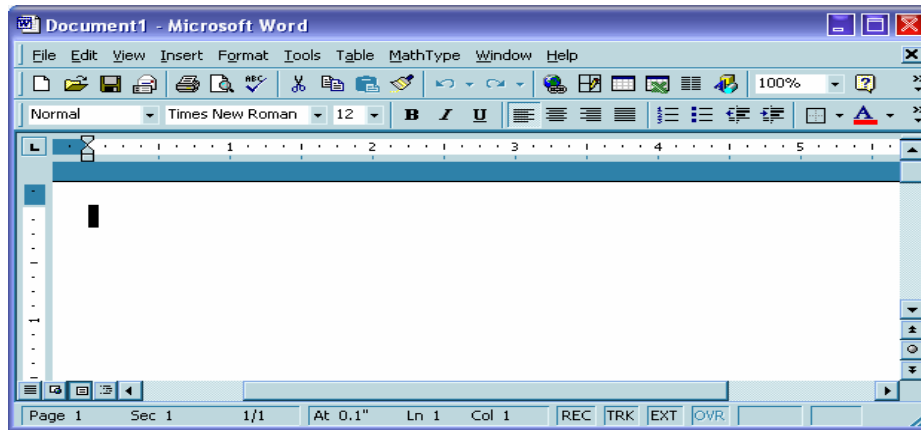
- a. Cantor was a German mathematician.
- b. Between 1874 and 1895 he developed the first clear and comprehensive account of transfinite sets and numbers.
- c. He was born in 1845 in Germany.
- d. He provided a precise definition of an infinite set, distinguishing between those which were denumerable and those which were not.

• Listening

1. Pre-listening task

1.1 Study this word processing screen.

1. Menu bar
2. Insertion point
3. Status bar
4. Title bar
5. Formatting toolbar
6. Standard toolbar
7. Ruler



1.2 Can you identify the components above?

2. Listen to the tape and check your answers.

TRANSLATION

Translate into Vietnamese.

1. Burali-Forti's paradox:

A paradox stated by C. Burali-Forti in 1897. Every well-ordered set has an ordinal number, and, as the set of all ordinals is well ordered, it also has an ordinal number, say A . But the set of all ordinals up to and including a given ordinal, say B , is itself well ordered and has ordinal number $B + 1$. So the set of all ordinals up to and including A has ordinal number $A + 1$, which is greater than A , so that A both is and is not the ordinal number of all ordinals. This paradox is avoided in standard versions of set theory by denying that there exists a set of all ordinals.

2. Liar paradox:

The paradox that if someone says "I am lying", then if what is said is true then it is false, and if what is said is false then it is true. Traditionally it is thought to have been put forward in the 6th century BC by the Cretan philosopher Epimenides. The liar paradox is an example of a sentence that may be grammatically correct, yet logically self-contradictory.

WORD LIST

Here is a list of some of the words from the units of English for Mathematics.

Write the translation:

adj = adjective
conj = conjunction
pl = plural
pron = pronoun
n = noun
adv = adverb
opp = opposite

prep = preposition
pp = past participle
v = verb
phr.v = phrasal verb
phr.n = phrasal noun
idm = idiom
infml = informal

UNIT 1

access (v) : *truy cập, tra cứu*
addend (n) : *số hạng*
addition (n) : *phép cộng*
algebra (n) : *đại số*
algebraic (adj) : *(thuộc) đại số*
angle (n) : *góc*
Boolean algebra (n) : *đại số Bun*
broad-minded (adj) : *có tư tưởng rộng rãi, khoáng đạt*
calculus (n) : *phép tính*
community (n) : *cộng đồng*
distance (n) : *khoảng cách*
distance education : *giáo dục từ xa*
division (n) : *phép chia*
domain (n) : *miền, miền xác định*
dramatically (adv) : *(một cách) đột ngột*
equation (n) : *phương trình*
experiment (v) *thí nghiệm, thực nghiệm*
fundamental (n) : *nguyên tắc cơ bản*
generalize (v) : *tổng quát hoá*
gravitation (n) : *trọng lực*
homological algebra (n) : *đại số đồng điều*
Lie group (n) : *nhóm Li*
linear algebra (n) : *đại số tuyến tính*
matrix algebra (n) : *đại số ma trận*
minuend (n) : *số bị trừ*
multiplication (n) : *phép nhân*

notation (n) : *ký hiệu*
nucleus (n) : *hạch, hạt nhân*
penetration (n) : *sự thâm nhập, thấm vào*
probability (n) : *xác suất*
problem (n) : *bài toán, vấn đề*
product (n) : *tích số*
proxy (n) : *sự uỷ nhiệm, người được uỷ nhiệm*
quantum (n) : *lượng tử*
quotient (n) : *thương*
regard (n) : *sự quan tâm*
in this regard (idm) : *về mặt này*
remainder (n) = difference : *hiệu số*
research (v) & (n) : *nghiên cứu*
solar system (n) : *hệ mặt trời*
solve (v) : *giải*
straightedge (n) : *ê-ke*
subtle (adj) : *tinh tế*
subtraction (n) : *phép trừ*
subtrahend (n) : *số trừ*
sum (n) : *tổng*
synonymous (adj) : *có cùng nghĩa*
triangle (n) : *tam giác*
isosceles triangle : *tam giác cân*
up-to-date (adj) : *cập nhật*
vector algebra (n) : *đại số vectơ*

UNIT 2

associative (adj) : (có tính) *liên kết, kết hợp*
axiom (n) : *tiên đề*
bequeath (v) : *để lại*
bracket (n) : *dấu ngoặc*
commutative (adj) : (có tính) *giao hoán*
compatibility (n) *tính tương thích, thích hợp*
component (n) : *thành phần*
congruence (n) : *đồng dư, tương đẳng*

conjecture (v) & (n) : giả định, phỏng đoán
 corollary (n) : hệ luận, hệ quả
 customary (adj) : theo thông lệ
 differential (n) : vi phân
 distributive (adj) : (có tính) phân phối, phân bố
 equivalence (n) : sự tương đương
 equivalent (adj) – equivalently (adv)
 error (n) : sai số, độ sai
 formula (n) : công thức
 fraction (n) : phân số
 futile (adj) : vô ích, ngớ ngẩn
 geometry (n) : hình học
 indefinitely (adv) : không xác định được
 integer (n) : số nguyên
 interval (n) : khoảng, đoạn
 irrational number : số vô tỷ
 mainframe computer (n) máy tính lớn, công suất cao, có bộ nhớ rộng
 marginal (adj) : (thuộc bên lề)
 marginal note : ghi chú bên lề
 polygon (n) : đa giác
 regular polygon : đa giác đều
 prime (n) : số nguyên tố
 property (n) : tính chất, thuộc tính
 ratio (n) : tỉ số
 rational number : số hữu tỷ
 real number : số thực
 repute (v) – be reputed : cho là, đồn là
 satisfy (v) : thỏa mãn
 solution (n) : phép giải, nghiệm
 spring (n) : lò xo
 stimulus (n) : tác nhân kích thích
 subscript (n) : chỉ số dưới
 subsequent (adj) : (thuộc về) dãy con
 tarnish (v) : làm mờ
 theorem (n) : định lý

triple (n) : *bộ ba*
trisection (n) : *sự chia đều cho 3*
utilize (v) : *dùng*
verify (v) : *thử lại*

UNIT 3

abrupt (adj) : *bất ngờ, đột ngột*
accustomed (adj) to sth : *quen với*
arbitrary (adj) : *tùy ý*
asset (n) : *của cải, tài sản*
branch (n) : *nhánh*
collection (n) : *tập hợp*
combat (v) : *chống lại, đọ sức*
denominator (n) : *mẫu số*
denote (v) : *ký hiệu*
disclose (v) : *vạch ra*
dissertation (n) : *luận án, luận văn*
equivalent fraction : *phân số bằng nhau (phân số tương đương)*
extend (v) : *kéo dài, mở rộng*
gather up (phr.v) : *thu thập lại, tập trung*
identity (n) : *sự nhận biết, nhận diện, đồng nhất hoá*
imply (v) : *bao hàm, kéo theo, có nghĩa*
improper fraction : *phân số không thực*
induction (n) : *lượng hoá*
interpret (v) : *giải thích, thể hiện*
limit (n) : *ranh giới*
(v) : *giới hạn*
mixed fraction : *phân số hỗn hợp*
numeral (n) : *chữ số*
numerator (n) : *tử số*
operation (n) : *phép toán*
ordered field (n) : *trường được sắp*
pay off (phr.v) : *tranh trải, thanh toán*
phenomenon (n) – pl : *phenomena : hiện tượng*
plummet (v) : *rơi thẳng, tụt*

postulate (n) : *định đề, tiên đề*
procession (n) : *đoàn người, đám rước, đám diễu hành*
proof (n) : *sự chứng minh*
proper fraction : *phân số thực (sự)*
remarkable (adj) : *đáng lưu ý, khác thường*
set (n) : *tập hợp*
significant (adj) : *có ý nghĩa, đáng kể*
signify (v) : *có nghĩa là*
simple-minded (adj) : *chất phác, ngớ ngẩn*
stuck-up (adj) : *vênh váo, hợm hĩnh*
substantial (adj) *lớn lao, đáng kể*
substantially (adv) : *nhiều, đáng kể*
summation (n) : *phép cộng, phép lấy tổng, phép lấy tích phân*
symbol (n) : *ký hiệu, dấu*
synonymous (adj) : *có cùng nghĩa*
tend (to) (v) : *tiến đến, dẫn đến*

UNIT 4

a big shot (informal) : *nhân vật quan trọng*
bugler (n) : *người thổi kèn đồng*
category (n) : *phạm trù, hạng mục*
conventional (adj) : *theo quy ước, thông thường*
diminished (adj) : *(được) làm nhỏ lại, rút ngắn lại*
feature (n) : *nét, đặc điểm*
flag pole (n) : *cột cờ*
illustrate (v) : *minh họa*
inequality (n) : *bất đẳng thức*
absolute inequality : *bất đẳng thức tuyệt đối*
conditional inequality : *bất đẳng thức có điều kiện*
unconditional inequality : *bất đẳng thức vô điều kiện*
invincible (adj) : *vô địch*
positive number : *số dương*
range (n) : *khoảng biến thiên, giao độ, hàng, phạm vi, miền*
range of value : *miền giá trị*
reflexive (adj) : *phản xạ*

relation (n) : *quan hệ, hệ thức*
sentence (n) : *câu, mệnh đề*
closed sentence : *câu đóng*
sophisticated (adj) : *sành điệu, tinh vi*
statement (n) : *mệnh đề, phát biểu*
swagger (v) : *ngênh ngang, vênh váo*
symbol (n) : *kí hiệu, dấu*
symmetric (adj) : *đối xứng*
transitive (adj) : *bắc cầu, truyền ứng*
variable (n) : *biến số, biến thiên, biến đổi*

UNIT 5

advance (v) : *tiến bộ, thúc đẩy*
concern (v) : *liên quan (đến)*
be concerned with : *đề cập đến ...*
corresponding (adj) : *tương ứng*
curve (n) : *đường cong*
define (v) : *định nghĩa, xác định*
derivative (n) : *đạo hàm*
dimension (n) : *kích thước, chiều, thứ nguyên*
display (v) : *bày ra, trưng bày, để lộ ra*
extend (v) : *kéo dài, mở rộng*
extract (v) : *rút ra, trích ra*
figure (n) : *hình vẽ, biểu đồ, ký hiệu*
function (n) : *hàm số*
indeterminate (adj) : *bất định, vô định*
intersect (v) : *cắt, giao nhau*
intersection (n) : *chỗ giao nhau*
intriguing (adj) : *hấp dẫn*
locus (n) : *quỹ tích, vị trí*
master (v) : *nắm vững, tinh thông*
misleading (adj) : *sai, lừa dối*
mysterious (adj) : *huyền bí, bí ẩn*
polish up (phr.v) : *đánh bóng, trau chuốt*
projective geometry : *hình học xạ ảnh*

ratio (n) : *tỉ số, tỉ suất*
ray (n) : *tia, nửa đường thẳng*
reason (v) : *suy luận*
reasoning (n) : *sự lập luận, tranh luận*
segment (n) : *đoạn*
sequence (n) : *dãy*
store (v) : *lưu trữ*
subject (n) : *môn học*
subset (n) : *tập hợp con*
tangent (n) : *tiếp tuyến, tang*
vertex (n) : *đỉnh*
volume (n) : *khối, thể tích*

UNIT 6

accommodate (v) : *cung cấp, xem xét, điều chỉnh*
admit (v) : *thú nhận*
area (n) : *diện tích*
compute (v) : *tính toán bằng máy tính, tính toán, suy tính*
congruent (adj) : *đồng dạng*
credit (v) : *tin, công nhận, ghi vào*
curious (adj) : *muốn tìm hiểu, tò mò, khác thường*
dashed (adj) : *(được) nhấn mạnh, gạch nét*
executive (n) : *ủy viên ban quản trị*
fascinating (adj) : *có sức hấp dẫn, quyến rũ*
financial (adj) (thuộc về) *tài chính*
hypotenuse (n) : *cạnh huyền*
interior (n) : *phần bên trong*
manuscript (n) : *bản viết tay, bản thảo*
motivate (v) : *thúc đẩy*
mystical (adj) : *thần bí, huyền bí*
operation (n) : *sự hoạt động*
pictorial (adj) : *được minh họa, có tranh ảnh*
prospective (adj) : *về sau, sắp tới*
radix (n) : *cơ số*

resemble (v) : *giống, tương tự*
right angle : *góc vuông*
seminar (n) : *hội thảo chuyên đề*
stable (adj) : *bền vững, vững chắc*
straight angle : *góc bẹt*
stretch (v) : *kéo dài ra*
triangular (adj) : (thuộc) *tam giác*
union (n) : *hợp*
upgrade (v) : *nâng cấp*
wonder (n) : *điều kỳ diệu*

UNIT 7

accurately (adv) : *một cách chính xác*
analytic geometry (n) : *hình học giải tích*
axis (n) : *trục*
clarify (v) : *làm sáng tỏ*
communicate (v) : *truyền đạt, liên lạc với nhau*
compulsory (adj) : *bắt buộc, ép buộc, cưỡng bách*
concentrate (v) : *tập trung*
conform (v) : *giữ quy ước, tuân thủ, tương ứng, phù hợp*
cooperate (v) : *hợp tác*
coordinate (n) : *tọa độ*
coordinate plane : *mặt phẳng tọa độ*
correspond (v) : *tương ứng*
criticise (v) : *chỉ trích, phê bình*
desirable (adj) : *mong muốn*
determine (v) : *xác định*
effective (adj) : *hữu hiệu, có hiệu quả*
equivalent (n) : *số hoặc từ tương đương*
horizontal line (n) : *đường nằm ngang*
incisiveness (n) : *sự sắc sảo*
infinity (n) : (sự) *vô tận, vô cùng lớn*
intentional (adj) : *có ý, có chủ tâm*
intersect (v) : *cắt, giao nhau*

label (v) : *ký hiệu, đánh dấu*
locate (v) : *định vị trí, đặt*
memorize (v) : *ghi nhớ, học thuộc*
negation (n) : *sự phủ định*
negative number : *số âm*
non-communicative (adj) : *không giao hoán*
operate (v) : *vận hành, sử dụng, làm cho chuyển động*
ordered pairs : *cặp được sắp*
parallel (n) : *(sự) song song, đường song song*
perpendicular (adj) : *thẳng góc*
plane (n) : *mặt phẳng*
procedure (n) : *cách, biện pháp, phương pháp*
proceed (v) : *tiếp tục, phát sinh, xuất hiện*
product (n) : *tích, tích số*
Cartesian product : *tích Đề Các*
purpose (n) : *mục đích*
relate (v) : *liên quan*
simplified (adj) : *(có tính) đơn giản hoá*
subtract (v) : *trừ*
vector sum : *tổng của vectơ*
vertical line (n) : *đường thẳng đứng*
vice versa : *ngược lại*

UNIT 8

acute (adj) : *nhọn*
algorithm (n) : *thuật toán, an gô rít*
allocate (v) : *sắp xếp, phân bố*
antecedent (n) : *tiền lệ, tiền kiện*
banish (v) : *trục xuất, xua đuổi*
basis (n) : *cơ sở*
cone (n) : *hình nón*
conic (n) : *côníc, đường bậc hai*
consequent (n) : *hậu thức*
database (n) : *cơ sở dữ liệu*

datum (n) (pl. data) : số liệu, dữ kiện
diameter (n) : đường kính
digit (n) : chữ số, hàng số
digital (adj) : (thuộc) số
directrix (n) : đường chuẩn
eccentricity (n) : tâm sai, độ lệch tâm
elaborate (v) : chi tiết hoá, bổ sung chi tiết
ellipse (n) : elip
elongated (adj) : lỵ giác, (được) kéo dài ra, giãn ra
encounter (v) : gặp, chạm trán
evaluate (v) : đánh giá, ước lượng
evolution (n) : sự tiến hoá
exceed (v) : vượt trội
exhaustive (adj) : vét kiệt
extreme (n) : cực trị, cực hạn
field (n) : trường, miền, thể
focus (n) : tiêu điểm
frame of reference (phr.n) : hệ quy chiếu, hệ toạ độ
hyperbola (n) : hypebôn
logarithm table (n) : bảng lô ga
mean (n) : giá trị trung bình, phương tiện, phương pháp
mechanics (n) : cơ học
obtuse (adj) : tù (góc)
origin (n) : (nguồn) gốc, nguyên bản
parabola (n) : parabôn
physics (n) : vật lý học
proportion (n) : tỉ lệ, tỉ lệ thức
rectangular (adj) : (thuộc) hình chữ nhật
relevance (n) : sự liên quan
restriction (n) : sự hạn chế, giới hạn, thu hẹp
section (n) : tiết diện
conic section : tiết diện côníc
slide (n) : sự trượt, con trượt
solid (n) : cố thể
solid geometry (n) : hình học không gian

spirit (n) : *tinh thần*
submerge (v) : *dìm, nhện chìm*
symmetrical (adj) : *đối xứng*
term (n) : *số hạng*
transistor (n) : *(máy, đèn) bán dẫn*
transverse (adj) : *ngang*
treat (v) : *xử lý*
unsolvability (n) : *(sự, tính) không giải được*

UNIT 9

adjunct (n) : *bổ ngữ, vật phụ gia*
arithmetic (n) : *số học*
bus (n) : *xe*
carry out (phr.v) : *tiến hành, thực hiện*
coded : *(bằng) mật mã*
constant (adj) : *liên miên, không thay đổi, bất biến*
control (n) : *bộ điều chỉnh*
cosmetics (n) : *mỹ phẩm*
decimal digit (n) : *số thập phân*
demodulator (n) : *cái khử biến điện*
dice (n) : *con súc sắc*
dictate (v) : *ra lệnh*
equality (n) : *đẳng thức*
even (adj) : *bằng nhau, chẵn*
failure (n) : *sự hỏng, sự thất bại*
favor (favour) (v) : *thiên vị*
file (n) : *(máy tính) tệp*
function (n) : *hàm số*
head (n) : *mặt ngửa (of a coin)*
impetus (n) : *sự thúc đẩy*
input (n) : *đầu vào*
launch (v) : *phát động, tung ra*
locus (n) : *quỹ tích, vị trí*
magnitude (n) : *độ lớn, độ dài, chiều đo*

mesh (n) : (top) *độ nhỏ*
 modulator (n) : *máy biến điện*
 occurrence (n) : *sự xuất hiện*
 odds (n) : *lợi thế*
 outcome (n) : *hậu quả*
 output (n) : *đầu ra*
 polar (adj) : (*thuộc về*) *cực, cực tuyến, cực diện*
 probability (n) : *xác suất*
 purchase (v) : *mua, tậu*
 radius (n) : *bán kính*
 ring (n) : *vành*
 simultaneous (adj) : *đồng thời, cùng lúc*
 star (n) : *hình sao, dấu sao*
 sum (n) : *tổng*
 tail (n) : *mặt sấp (of a coin)*
 topology (n) : *tô pô*
 toss (v) : *ném, tung*
 trace (v) : *theo, lần ra*

WORD LIST

Here is a list of some of the words from the units of English for Mathematics.

Write the translation:

adj = adjective	prep = preposition
conj = conjunction	pp = past participle
pl = plural	v = verb
pron = pronoun	phr.v = phrasal verb
n = noun	phr.n = phrasal noun
adv = adverb	idm = idiom
opp = opposite	infml = informal

UNIT 1

access (v) : *truy cập, tra cứu*
 addend (n) : *số hạng*
 addition (n) : *phép cộng*

algebra (n) : *đại số*
algebraic (adj) : (thuộc) *đại số*
angle (n) : *góc*
Boolean algebra (n) : *đại số Bun*
broad-minded (adj) : *có tư tưởng rộng rãi, khoáng đạt*
calculus (n) : *phép tính*
community (n) : *cộng đồng*
distance (n) : *khoảng cách*
distance education : *giáo dục từ xa*
division (n) : *phép chia*
domain (n) : *miền, miền xác định*
dramatically (adv) : (một cách) *đột ngột*
equation (n) : *phương trình*
experiment (v) *thí nghiệm, thực nghiệm*
fundamental (n) : *nguyên tắc cơ bản*
generalize (v) : *tổng quát hoá*
gravitation (n) : *trọng lực*
homological algebra (n) : *đại số đồng điều*
Lie group (n) : *nhóm Li*
linear algebra (n) : *đại số tuyến tính*
matrix algebra (n) : *đại số ma trận*
minuend (n) : *số bị trừ*
multiplication (n) : *phép nhân*
notation (n) : *ký hiệu*
nucleus (n) : *hạch, hạt nhân*
penetration (n) : *sự thâm nhập, thấm vào*
probability (n) : *xác suất*
problem (n) : *bài toán, vấn đề*
product (n) : *tích số*
proxy (n) : *sự uỷ nhiệm, người được uỷ nhiệm*
quantum (n) : *lượng tử*
quotient (n) : *thương*
regard (n) : *sự quan tâm*
in this regard (idm) : *về mặt này*
remainder (n) = difference : *hiệu số*

research (v) & (n) : *ngiên cứu*
solar system (n) : *hệ mặt trời*
solve (v) : *giải*
straightedge (n) : *ê-ke*
subtle (adj) : *tinh tế*
subtraction (n) : *phép trừ*
subtrahend (n) : *số trừ*
sum (n) : *tổng*
synonymous (adj) : *có cùng nghĩa*
triangle (n) : *tam giác*
isosceles triangle : *tam giác cân*
up-to-date (adj) : *cập nhật*
vector algebra (n) : *đại số vectơ*

UNIT 2

associative (adj) : (có tính) *liên kết, kết hợp*
axiom (n) : *tiên đề*
bequeath (v) : *để lại*
bracket (n) : *dấu ngoặc*
commutative (adj) : (có tính) *giao hoán*
compatibility (n) *tính tương thích, thích hợp*
component (n) : *thành phần*
congruence (n) : *đồng dư, tương đẳng*
conjecture (v) & (n) : *giả định, phỏng đoán*
corollary (n) : *hệ luận, hệ quả*
customary (adj) : *theo thông lệ*
differential (n) : *vi phân*
distributive (adj) : (có tính) *phân phối, phân bố*
equivalence (n) : *sự tương đương*
equivalent (adj) – equivalently (adv)
error (n) : *sai số, độ sai*
formula (n) : *công thức*
fraction (n) : *phân số*
futile (adj) : *vô ích, ngớ ngẩn*
geometry (n) : *hình học*

indefinitely (adv) : *không xác định được*
integer (n) : *số nguyên*
interval (n) : *khoảng, đoạn*
irrational number : *số vô tỷ*
mainframe computer (n) *máy tính lớn, công suất cao, có bộ nhớ rộng*
marginal (adj) : *(thuộc bên lề)*
marginal note : *ghi chú bên lề*
polygon (n) : *đa giác*
regular polygon : *đa giác đều*
prime (n) : *số nguyên tố*
property (n) : *tính chất, thuộc tính*
ratio (n) : *tỉ số*
rational number : *số hữu tỷ*
real number : *số thực*
repute (v) – be reputed : *cho là, đồn là*
satisfy (v) : *thỏa mãn*
solution (n) : *phép giải, nghiệm*
spring (n) : *lò xo*
stimulus (n) : *tác nhân kích thích*
subscript (n) : *chỉ số dưới*
subsequent (adj) : *(thuộc về) dãy con*
tarnish (v) : *làm mờ*
theorem (n) : *định lý*
triple (n) : *bộ ba*
trisection (n) : *sự chia đều cho 3*
utilize (v) : *dùng*
verify (v) : *thử lại*

UNIT 3

abrupt (adj) : *bất ngờ, đột ngột*
accustomed (adj) to sth : *quen với*
arbitrary (adj) : *tùy ý*
asset (n) : *của cải, tài sản*
branch (n) : *nhánh*
collection (n) : *tập hợp*

combat (v) : *chống lại, đọ sức*
 denominator (n) : *mẫu số*
 denote (v) : *ký hiệu*
 disclose (v) : *vạch ra*
 dissertation (n) *luận án, luận văn*
 equivalent fraction : *phân số bằng nhau (phân số tương đương)*
 extend (v) : *kéo dài, mở rộng*
 gather up (phr.v) : *thu thập lại, tập trung*
 identity (n) : *sự nhận biết, nhận diện, đồng nhất hoá*
 imply (v) : *bao hàm, kéo theo, có nghĩa*
 improper fraction : *phân số không thực*
 induction (n) : *lượng hoá*
 interpret (v) : *giải thích, thể hiện*
 limit (n) : *ranh giới*
 (v) : *giới hạn*
 mixed fraction : *phân số hỗn hợp*
 numeral (n) : *chữ số*
 numerator (n) : *tử số*
 operation (n) : *phép toán*
 ordered field (n) : *trường được sắp*
 pay off (phr.v) *trả trái, thanh toán*
 phenomenon (n) – pl : phenomena : *hiện tượng*
 plummet (v) : *rơi thẳng, tụt*
 postulate (n) : *định đề, tiên đề*
 procession (n) : *đoàn người, đám rước, đám diễu hành*
 proof (n) : *sự chứng minh*
 proper fraction : *phân số thực (sự)*
 remarkable (adj) : *đáng lưu ý, khác thường*
 set (n) : *tập hợp*
 significant (adj) : *có ý nghĩa, đáng kể*
 signify (v) : *có nghĩa là*
 simple-minded (adj) : *chất phác, ngờ ngẩn*
 stuck-up (adj) : *vênh váo, hợm hĩnh*
 substantial (adj) *lớn lao, đáng kể*
 substantially (adv) : *nhiều, đáng kể*

summation (n) : *phép cộng, phép lấy tổng, phép lấy tích phân*

symbol (n) : *ký hiệu, dấu*

synonymous (adj) : *có cùng nghĩa*

tend (to) (v) : *tiến đến, dẫn đến*

UNIT 4

a big shot (informal) : *nhân vật quan trọng*

bugler (n) : *người thổi kèn đồng*

category (n) : *phạm trù, hạng mục*

conventional (adj) : *theo quy ước, thông thường*

diminished (adj) : *(được) làm nhỏ lại, rút ngắn lại*

feature (n) : *nét, đặc điểm*

flag pole (n) : *cột cờ*

illustrate (v) : *minh họa*

inequality (n) : *bất đẳng thức*

absolute inequality : *bất đẳng thức tuyệt đối*

conditional inequality : *bất đẳng thức có điều kiện*

unconditional inequality : *bất đẳng thức vô điều kiện*

invincible (adj) : *vô địch*

positive number : *số dương*

range (n) : *khoảng biến thiên, giao độ, hàng, phạm vi, miền*

range of value : *miền giá trị*

reflexive (adj) : *phản xạ*

relation (n) : *quan hệ, hệ thức*

sentence (n) : *câu, mệnh đề*

closed sentence : *câu đóng*

sophisticated (adj) : *sành điệu, tinh vi*

statement (n) : *mệnh đề, phát biểu*

swagger (v) : *ngênh ngang, vênh váo*

symbol (n) : *kí hiệu, dấu*

symmetric (adj) : *đối xứng*

transitive (adj) : *bắc cầu, truyền ứng*

variable (n) : *biến số, biến thiên, biến đổi*

UNIT 5

- advance (v) : *tiến bộ, thúc đẩy*
concern (v) : *liên quan (đến)*
be concerned with : *đề cập đến ...*
corresponding (adj) : *tương ứng*
curve (n) : *đường cong*
define (v) : *định nghĩa, xác định*
derivative (n) : *đạo hàm*
dimension (n) : *kích thước, chiều, thứ nguyên*
display (v) : *bày ra, trưng bày, để lộ ra*
extend (v) : *kéo dài, mở rộng*
extract (v) : *rút ra, trích ra*
figure (n) : *hình vẽ, biểu đồ, ký hiệu*
function (n) : *hàm số*
indeterminate (adj) : *bất định, vô định*
intersect (v) : *cắt, giao nhau*
intersection (n) : *chỗ giao nhau*
intriguing (adj) : *hấp dẫn*
locus (n) : *quỹ tích, vị trí*
master (v) : *nắm vững, tinh thông*
misleading (adj) : *sai, lừa dối*
mysterious (adj) : *huyền bí, bí ẩn*
polish up (phr.v) : *đánh bóng, trau chuốt*
projective geometry : *hình học xạ ảnh*
ratio (n) : *tỉ số, tỉ suất*
ray (n) : *tia, nửa đường thẳng*
reason (v) : *suy luận*
reasoning (n) : *sự lập luận, tranh luận*
segment (n) : *đoạn*
sequence (n) : *dãy*
store (v) : *lưu trữ*
subject (n) : *môn học*
subset (n) : *tập hợp con*
tangent (n) : *tiếp tuyến, tang*

vertex (n) : *đỉnh*

volume (n) : *khối, thể tích*

UNIT 6

accommodate (v) : *cung cấp, xem xét, điều chỉnh*

admit (v) : *thú nhận*

area (n) : *diện tích*

compute (v) : *tính toán bằng máy tính, tính toán, suy tính*

congruent (adj) : *đồng dạng*

credit (v) : *tin, công nhận, ghi vào*

curious (adj) : *muốn tìm hiểu, tò mò, khác thường*

dashed (adj) : *(được) nhấn mạnh, gạch nét*

executive (n) : *ủy viên ban quản trị*

fascinating (adj) : *có sức hấp dẫn, quyến rũ*

financial (adj) (thuộc về) *tài chính*

hypotenuse (n) : *cạnh huyền*

interior (n) : *phần bên trong*

manuscript (n) : *bản viết tay, bản thảo*

motivate (v) : *thúc đẩy*

mystical (adj) : *thần bí, huyền bí*

operation (n) : *sự hoạt động*

pictorial (adj) : *được minh họa, có tranh ảnh*

prospective (adj) : *về sau, sắp tới*

radix (n) : *cơ số*

resemble (v) : *giống, tương tự*

right angle : *góc vuông*

seminar (n) : *hội thảo chuyên đề*

stable (adj) : *bền vững, vững chắc*

straight angle : *góc bẹt*

stretch (v) : *kéo dài ra*

triangular (adj) : (thuộc) *tam giác*

union (n) : *hợp*

upgrade (v) : *nâng cấp*

wonder (n) : *điều kỳ diệu*

UNIT 7

- accurately (adv) : *một cách chính xác*
analytic geometry (n) : *hình học giải tích*
axis (n) : *trục*
clarify (v) : *làm sáng tỏ*
communicate (v) : *truyền đạt, liên lạc với nhau*
compulsory (adj) : *bắt buộc, ép buộc, cưỡng bách*
concentrate (v) : *tập trung*
conform (v) : *giữ quy ước, tuân thủ, tương ứng, phù hợp*
cooperate (v) : *hợp tác*
coordinate (n) : *tọa độ*
coordinate plane : *mặt phẳng tọa độ*
correspond (v) : *tương ứng*
criticise (v) : *chỉ trích, phê bình*
desirable (adj) : *mong muốn*
determine (v) : *xác định*
effective (adj) : *hữu hiệu, có hiệu quả*
equivalent (n) : *số hoặc từ tương đương*
horizontal line (n) : *đường nằm ngang*
incisiveness (n) : *sự sắc sảo*
infinity (n) : *(sự) vô tận, vô cùng lớn*
intentional (adj) : *có ý, có chủ tâm*
intersect (v) : *cắt, giao nhau*
label (v) : *ký hiệu, đánh dấu*
locate (v) : *định vị trí, đặt*
memorize (v) : *ghi nhớ, học thuộc*
negation (n) : *sự phủ định*
negative number : *số âm*
non-communicative (adj) : *không giao hoán*
operate (v) : *vận hành, sử dụng, làm cho chuyển động*
ordered pairs : *cặp được sắp*
parallel (n) : *(sự) song song, đường song song*
perpendicular (adj) : *thẳng góc*

plane (n) : *mặt phẳng*
procedure (n) : *cách, biện pháp, phương pháp*
proceed (v) : *tiếp tục, phát sinh, xuất hiện*
product (n) : *tích, tích số*
Cartesian product : *tích Đề Các*
purpose (n) : *mục đích*
relate (v) : *liên quan*
simplified (adj) : *(có tính) đơn giản hoá*
subtract (v) : *trừ*
vector sum : *tổng của vectơ*
vertical line (n) : *đường thẳng đứng*
vice versa : *ngược lại*

UNIT 8

acute (adj) : *nhọn*
algorithm (n) : *thuật toán, an gô rít*
allocate (v) : *sắp xếp, phân bố*
antecedent (n) : *tiền lệ, tiền kiện*
banish (v) : *trục xuất, xua đuổi*
basis (n) : *cơ sở*
cone (n) : *hình nón*
conic (n) : *côníc, đường bậc hai*
consequent (n) : *hậu thức*
database (n) : *cơ sở dữ liệu*
datum (n) (pl. data) : *số liệu, dữ kiện*
diameter (n) : *đường kính*
digit (n) : *chữ số, hàng số*
digital (adj) : *(thuộc) số*
directrix (n) : *đường chuẩn*
eccentricity (n) : *tâm sai, độ lệch tâm*
elaborate (v) : *chi tiết hoá, bổ sung chi tiết*
ellipse (n) : *elíp*
elongated (adj) : *ly giác, (được) kéo dài ra, giãn ra*
encounter (v) : *gặp, chạm trán*

evaluate (v) : *đánh giá, ước lượng*
evolution (n) : *sự tiến hoá*
exceed (v) : *vượt trội*
exhaustive (adj) : *vết kiệt*
extreme (n) : *cực trị, cực hạn*
field (n) : *trường, miền, thể*
focus (n) : *trọng tâm*
frame of reference (phr.n) : *hệ quy chiếu, hệ tọa độ*
hyperbola (n) : *hypebôn*
logarithm table (n) : *bảng lô ga*
mean (n) : *giá trị trung bình, phương tiện, phương pháp*
mechanics (n) : *cơ học*
obtuse (adj) : *tù (góc)*
origin (n) : *(nguồn) gốc, nguyên bản*
parabola (n) : *parabôn*
physics (n) : *vật lý học*
proportion (n) : *tỉ lệ, tỉ lệ thức*
rectangular (adj) : *(thuộc) hình chữ nhật*
relevance (n) : *sự liên quan*
restriction (n) : *sự hạn chế, giới hạn, thu hẹp*
section (n) : *tiết diện*
conic section : *tiết diện côníc*
slide (n) : *sự trượt, con trượt*
solid (n) : *cố thể*
solid geometry (n) : *hình học không gian*
spirit (n) : *tinh thần*
submerge (v) : *dìm, nhậm chìm*
symmetrical (adj) : *đối xứng*
term (n) : *số hạng*
transistor (n) : *(máy, đèn) bán dẫn*
transverse (adj) : *ngang*
treat (v) : *xử lý*
unsolvability (n) : *(sự, tính) không giải được*

UNIT 9

- adjunct (n) : *bổ ngữ, vật phụ gia*
arithmetic (n) : *số học*
bus (n) : *xe*
carry out (phr.v) : *tiến hành, thực hiện*
coded : *(bằng) mật mã*
constant (adj) : *liên miên, không thay đổi, bất biến*
control (n) : *bộ điều chỉnh*
cosmetics (n) : *mỹ phẩm*
decimal digit (n) : *số thập phân*
demodulator (n) : *cái khử biến điện*
dice (n) : *con súc sắc*
dictate (v) : *ra lệnh*
equality (n) : *đẳng thức*
even (adj) : *bằng nhau, chẵn*
failure (n) : *sự hỏng, sự thất bại*
favor (favour) (v) : *thiên vị*
file (n) : *(máy tính) tệp*
function (n) : *hàm số*
head (n) : *mặt ngửa (of a coin)*
impetus (n) : *sự thúc đẩy*
input (n) : *đầu vào*
launch (v) : *phát động, tung ra*
locus (n) : *quỹ tích, vị trí*
magnitude (n) : *độ lớn, độ dài, chiều đo*
mesh (n) : *(top) độ nhỏ*
modulator (n) : *máy biến điện*
occurrence (n) : *sự xuất hiện*
odds (n) : *lợi thế*
outcome (n) : *hậu quả*
output (n) : *đầu ra*
polar (adj) : *(thuộc về) cực, cực tuyến, cực diện*
probability (n) : *xác suất*
purchase (v) : *mua, tậu*

radius (n) : *bán kính*
ring (n) : *vành*
simultaneous (adj) : *đồng thời, cùng lúc*
star (n) : *hình sao, dấu sao*
sum (n) : *tổng*
tail (n) : *mặt sấp (of a coin)*
topology (n) : *tô pô*
toss (v) : *ném, tung*
trace (v) : *theo, lần ra*

UNIT 10

alternatively (adv) : *(như một sự) lựa chọn*
appealing (adj) : *hấp dẫn, quyến rũ*
back and forth (idm) : *từ chỗ này đến chỗ kia, tới lui*
biunique (adj) : *một đối một*
chessboard (n) : *bàn cờ*
chord (n) : *(dây) cung, dây trướng*
coincidence (n) : *sự trùng hợp*
console (n) : *bảng điều khiển*
criterion (n) : *tiêu chuẩn*
curriculum (n) : *môn học*
debug a code (phr.v) : *chỉnh mã*
deform (v) : *làm biến dạng*
deformation (n) : *sự biến dạng*
dimensional (adj) : *(thuộc) chiều, thứ nguyên*
discard (v) : *thải bỏ*
distinct (adj) : *khác biệt, phân biệt rõ ràng*
drastic (adj) : *quyết liệt, nghiêm trọng*
equilibrium (n) : *sự cân bằng*
exponent (n) : *số mũ*
genus (n) : *giống*
hence (adv) : *do đó, từ đó, như vậy*
intuitive (adj) : *trực giác*
momentum (n) : *động lượng, xung lượng*

multiplier (n) : *số nhân, nhân tử*
oscillate (v) : *dao động, rung động*
polyhedron (n) (pl. polyhedra) : *khối đa diện*
possess (v) : *sở hữu*
principal axis (n) : *trục chính*
progression (n) : *cấp số*
geometric progression (n) : *cấp số nhân*
quantum (n) : *lượng tử*
readily (adv) : *một cách dễ dàng*
regular (adj) : *đều*
rigid (adj) : *cứng, rắn*
rigorousness (n) : *tính nghiêm ngặt, khắt khe*
spectrum (n) : *quang phổ, hàm phổ*
standard (n) : *tiêu chuẩn, mẫu*
stratum (n) : *địa tầng, vỉa, thớ*
subtend (n) : *trương, nằm đối diện*
transformation (n) : *phép biến đổi, phép ánh xạ*
twist (v) : *xoắn, quẩn, leo lên, bện, vặn*
vacuum (n) : *chân không*
virus (n) : *vi-rút*

UNIT 11

accompany (v) : *đi kèm theo, hộ tống*
analyse (v) : *phân tích*
arithmetic progression (n) : *cấp số cộng*
ashamed (adj) : *(tự) xấu hổ*
Banach space : *không gian Banach*
bidual (n) : *sự, phép song đối ngẫu*
bitranspose (n) : *liên hợp thứ, song chuyển vị*
bounded (adj) : *bị chặn*
characterize (v) : *mô tả, phát hoạ, tiêu biểu cho*
closed vector (n) : *vec-tơ đóng*
compact (n) : *compact*
continuous (adj) : *liên tục*

convex (adj) : *lồi, vồ lồi*
definite (adj) : *xác định*
demonstrate (v) : *chứng minh*
do business (phr.v) : *kinh doanh, thương mại, làm ăn*
dual (n) : *sự, phép đối ngẫu*
exclaim (v) : *kêu lên, la to (vì giận dữ)*
finite-dimensional (adj) : *hữu hạn chiều*
isomorphism (n) : *phép đẳng cấu*
finite (adj) : *hữu hạn*
idiot (n) : *người ngu ngốc, khờ khạo*
image (n) : *ảnh, hình ảnh*
indefinite (adj) : *không xác định*
infinite (adj) : *vô hạn*
install (v) : *lắp, đặt*
interlace (v) : *đan nhau, gắn với nhau*
mapping (n) : *ánh xạ*
mental (adj) : *(thuộc) tâm thần, tâm lý, trí tuệ*
modification (n) : *sự sửa đổi*
nilpotent (adj) : *lũy tính*
norm (n) : *chuẩn, sự định chuẩn*
observation (n) : *sự quan sát*
rely (on) (v) : *trông mong, dựa vào*
root (n) : *căn, nghiệm*
separated (adj) : *tách biệt*
speculation (n) : *sự suy đoán*
successive (adj) : *kế tiếp, liên tiếp*
transpose (n) : *liên hợp, chuyển vị*
unending (adj) : *bất tận*
visualize (v) : *hình dung, mừng tượng*
worth (adj) : *có giá trị, đáng giá*
yield (v) : *mang lại, sinh ra*

UNIT 12

abacus (n) : *bàn tính*
academy (n) : *học viện*

algebraic invariant : *bất biến đại số*
 animation (n) : *kỹ thuật làm phim hoạt hình*
 appoint (v) : *bổ nhiệm, cử*
 binary (adj) : *nhị nguyên, hai ngôi*
 coefficient (n) : *hệ số*
 collaboration (n) : *sự cộng tác*
 combination (n) : *(sự) phối hợp, tổ hợp*
 derivative (n) : *đạo hàm*
 designate (v) : *xác định, chỉ, ký hiệu*
 determinant (n) : *định thức*
 dilatation (n) : *phép giãn, sự giãn*
 equidistant (adj) : *cách đều*
 flash (v) : *loé lên, vụt qua*
 geneticist (n) : *nhà di truyền học*
 hacker (n) : *người ham thích lập trình, sử dụng máy vi tính
 người lấy trộm dữ liệu máy tính*
 hypothesis (n) : *giả thiết*
 implicit (adj) : *ngấm ngầm*
 index (n) : *chỉ số, cấp, bảng tra chữ cái*
 interminate (adj) : *bất định, vô định*
 invariant (n) : *(sự) bất biến*
 isolated (adj) : *bị cô lập*
 linear equation (n) : *phương trình tuyến tính*
 matrix (n) : *ma trận*
 media (n) : *phương tiện truyền thông đại chúng*
 motivation (n) : *động cơ thúc đẩy*
 parenthesis (n) : *dấu ngoặc đơn*
 permutation (n) : *sự hoán vị*
 polynomial (n) : *đa thức*
 potentially (adv) : *có khả năng, tiềm năng*
 prevalent (adj) : *phổ biến*
 reflection (n) : *sự phản xạ, sự đối xứng*
 rotation (n) : *phép quay, sự quay*
 scalar (adj) : *vô hướng*
 scenario (n) : *kịch bản, cốt truyện*
 statistician (n) : *nhà tĩnh học*
 synthesis (n) : *(phép, sự) tổng hợp*
 thesis (n) : *luận đề, thuyết, luận điểm*
 transposition (n) : *sự chuyển vị, sự đổi vế*
 undoubtedly (adv) : *không nghi ngờ, rõ ràng, chắc chắn*
 vanish (v) : *triệt tiêu, biến mất*

version (n) : *kiểu, bảng phóng tác*

UNIT 13

accomplish (v) : *thành công, hoàn thành tốt*

accuracy (n) : *(sự, độ) chính xác*

acquire (v) : *thụ đắc, dần dần có được*

application (n) : *sự ứng dụng*

astronomy (n) : *thiên văn học*

attraction (n) : *(sự, lực) hấp dẫn*

concept (n) : *khái niệm*

confidential (adj) : *tin cẩn, tin cậy*

constant function (n) : *hàm bất biến (hàm hằng)*

contribution (n) : *sự đóng góp*

deliver (v) : *phát, giao*

derivative (n) : *đạo hàm*

diagram (n) : *biểu đồ, sơ đồ*

distinguish (v) : *phân biệt*

duration (n) : *kéo dài, tiếp diễn*

emphasize (v) : *nhấn mạnh*

enable (v) : *làm cho (ai) có khả năng, có quyền*

for instance : *for example*

foundation (n) : *(sự) thành lập*

furnish (v) : *trang bị*

graph (n) : *biểu đồ, đồ thị*

heavenly (adj) : *thần tiên, thuộc về bầu trời*

heavenly bodies : *các thiên thể*

identity function (n) : *hàm đồng nhất*

inverse function (n) : *hàm ngược*

linear function (n) : *hàm tuyến tính*

obtain (v) : *có được*

one-to-one function (n) : *hàm một-một*

overestimate (v) : *đánh giá quá cao*

plot (v) : *vẽ, đánh dấu, vẽ đường đồ thị*

poverty (n) : *cảnh nghèo túng*

predict (v) : *dự đoán*

publish (v) : *xuất bản*

quaternion (n) : *quaternion*

quintic (adj) : *bậc năm, hạng năm*

rational function (n) : *hàm hữu tỷ*

recommend (v) : *giới thiệu, gợi ý*

revolutionize (v) : cách mạng hoá
rewind (v) : cuộn lại, quấn lại, quay lại
sketch (v) : vẽ phác hoạ
(n) : bức phác hoạ
slope (n) : độ dốc, độ nghiêng
smooth (adj) : trơn, nhẵn
solvability (n) : (tính, sự) giải được
source (n) : nguồn
stable (adj) : ổn định, dừng
status (n) : địa phận
stimulate (v) : khuấy động, kích thích
supply (v) : cung cấp, tiếp tế
take its toll (idm) : gây ra sự mất mát, thiệt hại
theoretical (adj) : liên quan hoặc thuộc về lý thuyết
thus (adv) : theo cách đó, như thế, như vậy
tuberculosis (n) : bệnh lao

UNIT 14

absurd (adj) : vô nghĩa, vô lý, phi lý
acceptable (adj) : có thể chấp nhận
affect (v) : ảnh hưởng, tác động
affirm (v) : tuyên bố, khẳng định
aggressive (adj) : hung hăng, hay gây sự
amateur (n) : người chơi nghiệp dư, tài tử
apparatus (n) : máy móc, công cụ
argument (n) : (sự) lý luận, chứng minh
beset (adj) : bị bao trùm
bush (n) : cây bụi, cây mọc thấp
carry on (phr.v) : tiếp tục
conception (n) : khái niệm, quan niệm
contradictory (adj) : mâu thuẫn, trái ngược
corona (n) : tán, quầng, hào quang
crisis (n) : khủng hoảng
deny (v) : phủ định
divisibility (n) : tính chia hết
formulation (n) : kết quả của việc trên
headquarters (n) : sở chỉ huy, cơ quan đầu não
instant (n) : lúc, khoảnh khắc, thời điểm
intervene (v) : chen vào hoặc ở giữa

lacuna (n) : *kẽ hở, chỗ thiếu*
 liar (n) : *kẻ nói dối*
 market research organization (phr.n) : *tổ chức nghiên cứu thị trường*
 modernism (n) : *chủ nghĩa hiện đại*
 modulus (n) : *môđun, giá trị tuyệt đối*
 motto (n) : *phương châm, khẩu hiệu*
 nebula (n) : *tinh vân*
 occupy (v) : *chiếm, giữ*
 overtake (v) : *vượt lên*
 paradox (n) : *ngịch lý*
 perception (n) : *sự nhận thức*
 portray (v) : *miêu tả sinh động, đóng vai, vẽ chân dung*
 poser (n) : *vấn đề hóc búa*
 rest (n) : *(sự) tĩnh, nghỉ*
 reveal (v) : *bộc lộ, để lộ ra*
 rhombus (n) : *hình thoi*
 startling (adj) : *ngạc nhiên, sững sốt, đáng chú ý*
 thereafter (adv) : *sau đó*
 tortoise (n) : *con rùa*
 traverse (v) : *đi ngang qua*
 violence (n) : *bạo lực*
 virtual (adj) : *thực sự*

Appendix

$\{a, b\}$	the set whose elements (or members) are a and b
$A = B$	A is equal to, or equals, B
$A \sim B$	A is equivalent to B
$A \subseteq B$	A is a subset of B
$A \subset B$	A is a proper subset of B
\emptyset	the null, or empty, set
$a \in A$	a is an element of, or is a member of, A
\emptyset, \notin , etc.	is not a subset of, is not an element of, etc.
$\{x \mid \dots\}$	the set of all x such that ...
$A \cup B$	<i>the union of sets A and B</i>
$A \cap B$	the intersection of sets A and B
A'	the complement of set A
N	the set of natural numbers
$Z (J)$	the set of integers

	Q	<i>the set of rational numbers</i>
$R \setminus Q$ (H)		the set of irrational numbers
R		the set of real numbers
I		the set of imaginary numbers
C		the set of complex numbers
$-a$		the additive inverse, or reciprocal, of a
a^{-1} or $1/a$		the multiplicative inverse, or reciprocal, of a
R_-		the set of negative real numbers
R_+		the set of positive real numbers
$a < b$		a is less than b
$a > b$		a is greater than b
$a \leq b$		a is less than or equal to b
$a \geq b$		a is greater than or equal to b
$ a $		the absolute value of a
x^n		the n th power of x , or x to the n th power
$P(x), D(y)$, etc.		P of x , D of y , etc.
$a^{1/n}$		the n th real root of a for $a \in R$, or the positive one if there are two
$\sqrt[n]{a}$		the n th real root of a for $a \in R$, or the positive one if there are two
(x, y)		the ordered pair of numbers whose first component is x and whose second component is y
$A \times B$		the Cartesian product of A and B
$R \times R$, or R^2		the Cartesian product of R and R
f, g, h, F , etc.		names of functions
$f(x)$		f of x , or value of f at x
f^{-1}		the inverse function of f
d		distance between two points
m		the slope of a line
(x, y, z)		the ordered triple of numbers whose first component is x , second component is y , and third component is z
$(R \times R) \times R$, or R^3		the Cartesian product of $(R \times R)$ and R
$\log_b x$		the logarithm to the base b of x
e		an irrational number, approximately equal to 2.7182818
$\cos x$		element in the range of the cosine function
$\sin x$		element in the range of the sine function
\bar{s} or \bar{x}		length of reference arc corresponding to arc of length s or x
$\operatorname{tg} x$ ($\tan x$)		element in the range of the tangent function

$\sec x$	element in the range of the secant function
$\operatorname{cosec} x$ ($\csc x$)	element in the range of the cosecant function
$\operatorname{cotg} x$ ($\cot x$)	element in the range of the cotangent function
$\alpha \cong \beta$	α is congruent to β
\overline{AB}	the ray AB
$m^\circ(\alpha)$	the measure of α in degree units
$m^R(\alpha)$	the measure of α in radian units
x°	x degrees
x^R (x rad)	x radians
\overline{AB}	the line segment AB
$l(\overline{AB})$	the length of \overline{AB}
A	area
$\operatorname{Sin}^{-1} x, \operatorname{Cos}^{-1} x,$ etc.	elements in the range of the principle-valued inverse circular or trigonometric functions.
$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix},$ etc.	matrix
$A_{m \times n}$	$m \times n$ matrix
$a_{i,j}$	the element in the i th row and j th column of matrix A
A^t	the transpose of matrix A
$\mathbf{0}_{m \times n}$	the $m \times n$ zero matrix
$-A_{m \times n}$	the negative of the matrix $A_{m \times n}$
$I_{n \times n}$	the identity matrix for all $n \times n$ matrices
$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix},$ etc.	determinant
$M_{i,j}$	the minor of the element $a_{i,j}$
$A_{i,j}$	the cofactor of $a_{i,j}$
$\det A$ ($\delta(A)$)	the determinant of A
A^{-1}	the inverse of A
$\left[\begin{array}{cc c} a_{11} & b_{12} & c_1 \\ a_{21} & b_{22} & c_2 \end{array} \right],$ etc.	the augmented matrix of $\begin{bmatrix} a_{11} & b_{12} \\ a_{21} & b_{22} \end{bmatrix},$ etc.
$A \sim B$	A is equivalent to B (for matrices)
z	complex number
$-z$	the additive inverse (or negative) of z
z^{-1}	the multiplicative inverse (or reciprocal) of z
\bar{z}	the conjugate of z
i	the imaginary unit, (0, 1)
$e^{i\theta}$ ($\operatorname{cis} \theta$)	$\cos \theta + i \sin \theta$
v	vector

$\ \mathbf{v}\ $	the norm, or magnitude, of \mathbf{v}
$\mathbf{0}$	the zero vector
$-\mathbf{v}$	the negative of \mathbf{v}
x	the unit vector $(1, 0)$ in the x -direction
y	the unit vector $(0, 1)$ in the y -direction
$s(n)$, or s_n	the n th term of a sequence
S_n	the sum of the first n terms in a sequence
$3S_\infty$	the sum of an infinite sequence
Σ	the sum
L	the limit of a sequence
$0.21\overline{21}$, etc.	repeating decimal
$0!$	Zero factorial
$n!$	n factorial, or factorial n
$n(A)$	the number of elements in the set A
$P_{n,n}$	the number of permutations of n things taken n at a time
$P_{n,r}$	the number of permutations of n things taken r at a time
$\binom{n}{r}$	the number of combinations of n things taken r at a time
E	event
$P(E)$	the probability of E
$P(E_2 E_1)$	the conditional probability of E_2 , given the occurrence of E_1

Glossary of Mathematical terms

abacus

Rods and beads used to show place values.

addend

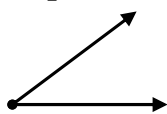
In an addition, the numbers used to name the sum.

addition

The renaming of two addends as a sum.

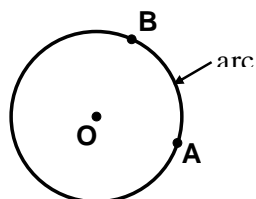
angle

The union of 2 rays with a common end point.



arc

Any part of a circle. AB is an arc.



area

The measure of a region together with the unit used for measurement.

Associative Property (of Addition or Multiplication)

The way in which the addends (or factors) are grouped does not affect the sum (or product).

$$a + (b + c) = (a + b) + c$$

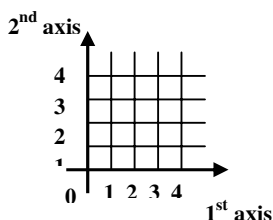
$$a \times (b \times c) = (a \times b) \times c$$

average

The average of a set of numbers is the number all the numbers would be if they were all the same and their sum did not change.

axis

One of the two number lines which form a number plane. The axes are usually called the *x*-axis and the *y*-axis of the first and the second axis.



bar graph

A graph that shows data by columns.

base

The number on which a place value system is built. The base of our system is ten.

base five numeration system

A numeration system using five as its base. The digits used are {0, 1, 2, 3, 4}.

base ten numeration system

A numeration system using ten as its base. The digits used are {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}.

basic fact

An addition or subtraction with addends less than 10, or a multiplication or division with factors less than 10.

braces { }

Symbol used to enclose elements in a set.

cardinal number

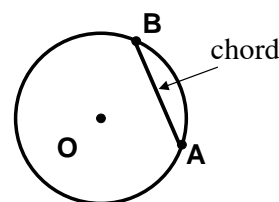
The number that tells how many objects are in a set.

center (of a circle)

Point in a circular region such that the distance from it to points on the circle is always the same.

chord (of a circle)

Any line segment that has both end points on the circle.



circle

A circle is a closed curve with all its points a fixed distance from a given point.

clock arithmetic

A system of numbers which may be shown on a circular number line.

closed curve

A curve that begins and ends at the same point.

closed surface

A surface which separates space into 3 sets of points, those inside, on, and outside the surface.

common factor

A number which is the factor of two or more other numbers.

common multiple

A number which is a multiple of two or more numbers.

Commutative Property
(of Addition or Multiplication)

The order of the addends (of factors) does not affect the sum (or product).

$$a + b = b + a$$

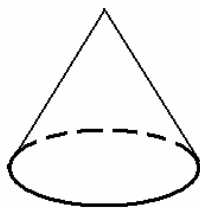
$$a \times b = b \times a$$

compact numeral

A numeral which makes full use of digits and place value, as 67.

cone

A closed surfaced formed by the union of a circular region and a curved surface and coming to a point at the top.



congruent

Having the same measure using the same unit.

counting numbers

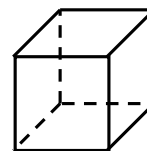
The numbers in $\{1, 2, 3, 4, \dots\}$.

cross product

The cross product of sets A and B , written $A \otimes B$, is the set of pairs with first members from A , and second members from B .

cube

A cube is a closed surface formed by the union of 6 congruent square regions with common sides.



cubic unit

A unit of volume used to measure the space inside a closed surface.

curve

A set of connected points that form a path.

cylinder

A closed surface formed by the union of two circular regions and a curved rectangular region.



data

A set of facts using numbers to show something.

decimal fraction

A numeral that uses place value and a decimal point to name a fractional number.

decimal point

In the decimal fraction .32, the point is called the decimal point.

degree

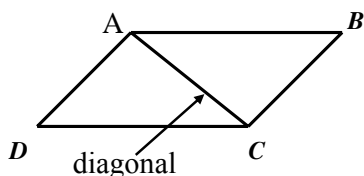
A standard unit for measuring angles. There are 360° (360 degrees) in a circle.

denominator

9 is the denominator of the fraction $\frac{2}{9}$. It names the number in the total set.

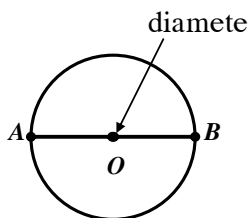
diagonal

A line segment in a polygon which joins 2 vertices but is not one of its sides. \overline{AC} is a diagonal of ABCD.



diameter

A diameter of a circle is any line segment that passes through the center, and has both end points on the circle. The diameter is length of such a line segment.



difference

The missing addend in a subtraction.

digit

A symbol used to write numerals. In our system of numeration the digits are 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

disjoint sets

Sets that have no common elements.

Distributive Property of Multiplication

To name the sum of two addends with a common factor, for example

$$(3 \times 5) + (4 + 5)$$

we may multiply, then add

$$(15 + 20 = 35)$$

or add, then multiply

$$(3 + 4 = 7, 7 \times 5 = 35).$$

divisible

Because $9 \times 3 = 27$, we say that 27 is divisible by 9 and by 3.

division

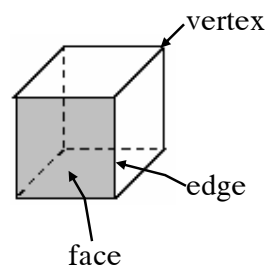
The renaming of a product and one factor as the other factor.

divisor

The given factor in a division.

edge

In certain space figures, the intersection of the flat surfaces (faces) of a simple closed surface.



ellipse

The intersection of a cylinder and a plane not parallel or perpendicular to the base of the cylinder.

empty set

The set that has no elements. Its cardinal number is zero.

equal sets

Sets that have exactly the same elements.

equation

A mathematical sentence stating that 2 numerals name the same number.

equivalent fractions

Fractions that name the same fractional number.

equivalent sets

Sets with the same cardinal number.

estimate

To say what you expect the answer of measurement to be before you do the work.

expanded numeral

A numeral such as

$$30 + 7 \text{ or } 7 + \frac{3}{4}.$$

exponent

A small numeral written above and to the right of another numeral. The exponent tells how many times the base number appears as a factor. For example, in 5^3 , 3 is the exponent; it shows 5 appears as a factor 3 times. $5^3 = 5 \times 5 \times 5$.

faces

(See “**edge**.”) The plane regions of a simple closed surface.

face value

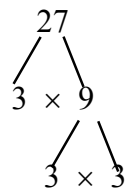
The value a digit always has. In 637 the face value of the 6 is 6.

factor

A number that is multiplied by another number to name a product. In the equation $4 \times 3 = 12$, 4 and 3 are factors.

factor tree

A tree-like arrangement of factors to show a prime factorization as



finite set

A set whose members can be listed and counted. {All the people in the world}, though very large, is finite.

fraction

A numeral naming a fractional number.

fractional number

A number used to compare a subset and a set.

function

A set of ordered pairs like

$$\{(0, 2), (1, 3), (2, 4), (3, 5), \dots\}.$$

In this function we have the rule “add 2”.

greatest Common Factor

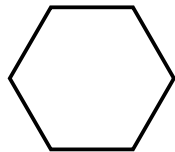
The greatest number in a set of common factors of two numbers.

graph

A picture used to show data. Different types of graphs are bar, line, and circle.

hexagon

A six-sided polygon.

**inequality**

A number sentence which states that one number is greater (or less) than another.

infinite set

A non-empty set which is not finite. All of its members cannot be listed and counted.

intersection (of sets)

The intersection of A and B , written $A \cap B$, is the set of all elements that are common to both A and B .

kilogram

A basic unit of measurement of weight in the metric system.

Lattice Method

A way of multiplying.

Least Common Multiple

The least nonzero member of the set of common multiples of two numbers.

line

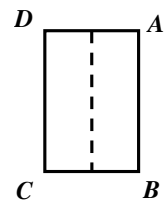
A straight curve that goes on indefinitely in two directions. A line is named by any 2 points on it. \overline{AB} is read "line AB ".

line segment

Part of a line with 2 end points. \overline{AB} is read "line segment AB ".

line of symmetry

Separates a curve into matching parts.

**liter**

A basic unit of measurement of capacity in the metric system.

measure

A number that tells how many units match an object for the property being measured: length, area, or volume.

median

The number which comes in the middle of a set of numbers when they are arranged in order.

meter

A basic unit of measurement of length in the metric system.

metric system

A system of measurement which uses ten as its base.

mixed numeral

A numeral with part naming a whole number and part naming a fractional number less than 1.

multiple

The multiple of a number is a product of that number and a whole number. The multiples of 3 may be found by multiplying 3 by the members of the set $\{0, 1, 2, 3, 4, \dots\}$.

multiplication

The renaming of two or more factors as a product.

number line

A line with points labeled by numbers.

number pattern

An arrangement of numbers according to a rule.

number plane

A plane with points labeled by numbers pairs.

numeral

A name for a number.

numerator

The numerator of the fraction $\frac{2}{9}$ is 2.

It names the cardinal number of a subset.

One Property

When one is a factor, the product and the other factor are the same.

Opposites Property

Subtraction is the opposite of addition, and division is the opposite of multiplication.

ordered pair

A pair of numbers in which the order is important. For example (3, 6) is not the same as (6, 3).

parallel lines

Two (or more) lines in the same plane that never intersect. The distance between 2 parallel lines is everywhere the same.

parallel planes

Planes which do not intersect.

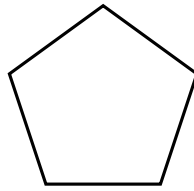
parallelogram

A quadrilateral with opposite sides parallel and congruent.



pentagon

A five-sided polygon.



perimeter

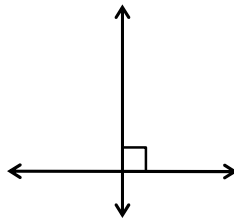
The distance around a plane figure.

periods

Grouping place values in hundreds, tens, and ones for ones, thousands, and millions periods.

perpendicular lines

Lines that intersect to form right angles (90°).



placeholder

A symbol holding a place for a numeral in a number sentence. In the number sentence $3 + 4 = n$, n is a placeholder.

place value

The value given to the place in which a digit appears. In 437, 4 is in the hundred's place, 3 is in the ten's place, and 7 is in the one's place.

plane

A flat surface that extends without end in all directions.

point

Undefined term in geometry. It may be thought of as the intersection of 2 lines, and represented by a dot.

polygon

A polygon is a simple closed curve formed as the union of line segments.

power

Any product shown by a base and an exponent. $3 \times 3 = 3^2$ is the second power of 3.

prime factorization

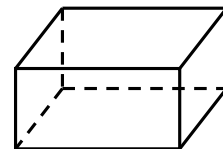
A numeral which names a number as a product of its prime factors.

prime number

A number with only 2 factors, itself and 1.

prism

A prism is a simple closed surface formed as the union of 6 rectangular regions.



probability

Comparing the chance of a particular thing happening with all the possibilities.

product

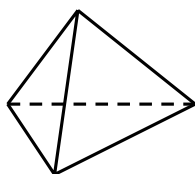
A number that results from renaming factors. In the equation $2 \times 5 = 10$, 10 is the product.

protractor

An instrument used for measuring and drawing angles. It is usually circular, and uses degrees as units.

pyramid

A pyramid is a simple closed surface made up of 3 or more triangular regions and a base.



quadrilateral

A four-sided polygon.

quotient

The number resulting from the division of one number by another.

radius

A radius of a circle is any line segment with one end point on the circle, the other the center of the circle. *The* radius of a circle is the length of such a line segment.

range

The difference between the greatest and least members of a set of data.

rate

A many-to-one matching of two sets with different members. Gary reads his book at the rate of 10 pages in 1 day.

ratio

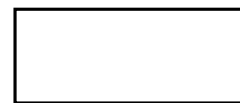
A fractional number used to compare two sets of like elements, one of which may be a subset of the other.

ray

Part of a line that has one end point and goes endlessly in one direction. \overrightarrow{AB} is read “ray *AB*” and *A* is the endpoint.

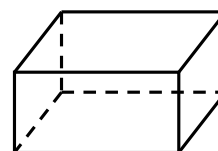
rectangle

A quadrilateral with 4 right angles.



rectangular prism

Closed surface formed by the union of three pairs of rectangular regions.



region

Points inside a closed curve.

regular polygon

A polygon with all its sides and all its angles congruent.

remainder

The number remaining in a division when the greatest multiple of the divisor has been subtracted from the dividend. 5 is the remainder in

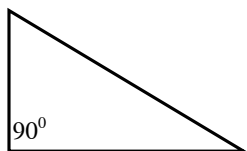
$$\begin{array}{r} 29 \overline{) 6} \\ 24 \\ \hline 5 \end{array}$$

right angle

The angle formed by the intersection of two perpendicular rays. A right angle has a measurement of 90° .

right triangle

A triangle which has one angle of 90° .

**scale drawing**

A similar but smaller or larger figure drawn using a ratio.

set

A collection of objects or ideas.

side

The line segment of a polygon; one of the two rays forming an angle; a plane region of a simple closed surface.

Sieve of Eratosthenes

A method of finding prime numbers discovered by the Greek mathematician, Eratosthenes.

similar polygons

Two polygons which have the same shape.

simple closed curve

A closed curve in a plane such that if you draw a picture of the curve, your pencil will return to the starting point, it will never leave the paper, and it will not go through any point twice.

size

The measure together with the unit of measure.

solution (of a number sentence)

A number whose name makes the sentence true.

sphere

A round closed surface, the points of which are a fixed distance from a fixed point.

square

A square is a quadrilateral with 4 right angles and 4 congruent sides.

**square unit**

A unit of area used to measure the area contained in a region.

subset

A is a subset of B if the members of A are all members of B .

subtraction

The renaming of a sum and an addend; the opposite of addition.

superset

B is a superset of A if A is a subset of B .

Surface Area

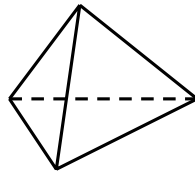
The combined area of all the faces of a closed surface.

symmetrical curve

A curve that can be separated into two matching parts.

tetrahedron

A closed surface formed by the union of four triangular regions.



Theorem of Pythagoras

The square of the measure of the longest side of a right triangle is equal to the sum of the squares of the measures of the others two sides. If c is the longest side, then

$$a^2 + b^2 = c^2.$$

total value

The product of the face and place values of a digit.

triangle

A simple closed curve formed as the union of three line segments. A polygon with 3 sides.



triangular prism

Closed surface formed by the union of 3 rectangular regions and a pair of triangular regions.

union

The union of A and B , written $A \cup B$, is the set of all the elements that are in A or in B .

universal set

The superset for sets we are talking about.

vertex

(See “Edge.”) The common end point of the 2 rays that form an angle or the point in which two sides of a polygon intersect.

volume

The measure of a closed surfaced together with units used for measurement.

whole numbers

The numbers in $\{0, 1, 2, 3, \dots\}$.

Zero Property

If zero is one addend, the sum and the other addend are the same.

Glossary of computing terms and abbreviations

access

Connect to, or get (information) from, a system or a database.

Active Server page

A type of webpage that contains a script that is processed on a web server.

adaptor board

A circuit board put in a spare slot in a microcomputer to control an external device.

address register

A register which stores an address in a memory.

ALGOL

Algorithmic language: a language developed for mathematical and scientific purposes.

algorithm

A prescribed set of well-defined rules or instructions for the solution to a problem.

analogue signal

A type of signal that can take any value between a maximum and a minimum.

arithmetic and logic unit

The part of the CPU that performs the mathematical and logical operations.

assembly language

A low-level computer language that uses mnemonics rather than only numbers, making it easier than machine code for humans to read and write.

backup device

A storage device used for copying files to a storage medium to keep them safe.

BASIC

Beginner's all-purpose symbolic instruction code: a programming language developed in the mid-1960s to exploit the capability (new at that time) of the interactive use of a computer from a terminal.

binary arithmetic

Arithmetic done to the base 2 using only 0 and 1 as its basic digits.

bookmark

A web address stored in a browser program to allow a webpage to be found easily / to store a web address in a browser program to allow a webpage to be found easily.

browser

A program used for displaying webpages.

bus topology

A physical layout of a network where all the computers are attached to one main cable terminated at both ends.

Byte

A unit of capacity. A byte is made up of eight bits and stores one character, i.e. a letter, a number, a space or a punctuation mark.

C

A highly portable programming language originally developed for the UNIX operating system, derived from BCPL via a short-lived predecessor B.

C + +

A programming language combining the power of object-oriented programming with the efficiency and notational convenience of C.

CALL

Computer Assisted Language Learning: the use of computers in the teaching of languages.

CD-ROM (disk)

Abbreviation for compact disk read-only storage device (a disk) that is read using laser light.

chip

Common name for a microchip.

click

To press and release a button on a mouse.

clipboard

See portable computer.

COBOL

Common business-oriented language: a high-level language designed for commercial business use.

code

A program written in a computer language / to write a program using a computer language.

command button

A dialog box component that takes the form of a rectangular icon that causes a program command to be carried out when clicked with a mouse.

compile

To convert a program written in a high-level language into machine code using a compiler.

compiler

A program that converts the whole of a program into machine code before the program is used.

computer

Put simply, a system that is capable of carrying out a sequence of operations in a distinctly and explicitly defined manner.

computer game

An interactive game played against a computer.

computerize

Provide a computer to do the work of / for something.

control unit

One of the two main components of the CPU. It transmits co-ordinating control signals and commands to the computer.

CPU

Central processing unit.

data

The information processed by a computer.

database

A type of application program used for storing information so that it can be easily searched and stored.

debug

To find and fix the faults in a program or system.

desktop (computer)

A personal computer designed to sit on a desk.

digital

The use of discrete digits to represent arithmetic numbers.

digital camera

An input device for taking pictures that has an electronic lens and uses electronics for storing the images rather than chemical film.

digital signal

A wave form or signal whose voltage at any particular time will be at any one of a group of discrete values (generally a two-level signal).

digitize

Convert analog signals to digital representation.

disk

A flat circular storage device.

disk drive

A storage device for reading from and writing to disks.

display

See **VDU**.

download

To copy a file from a server to a client computer in a network.

edit

To make changes to.

email

The common name for electronic mail, i.e. messages sent electronically using a computer / to send an email message.

email address

The unique address code used to contact someone using electronic mail.

execute

To perform a computer operation by processing a program instruction.

facsimile machine

A machine which will provide electronic transmission of documents over telephone lines.

fault-tolerant

Of a computer system, having the ability to recover from an error without crashing.

fibre-optic(s) cable

A cable made from strands of glass that is used for carrying information signals on a beam of light.

file

A computer program or data stored on a storage device.

folder

A way of grouping filenames so that the files can be easily located on a storage device. A folder is sometimes called a directory.

format (1)

The design and appearance of text in a document / to design the look of text in a document.

format (2)

The arrangement of storage areas on a storage medium / to create storage areas on a storage medium.

formatting toolbar

A row of icons in a program, used to change the appearance of the text when clicked with a mouse.

FORTRAN (77)

Formula translation: a programming language widely used for scientific computation. The '77' defines the year in which the official standard (to which the language conforms) was issued.

GB

Abbreviation for a gigabyte.

graphic

A picture, drawing, animation or other type of image.

hard (disk) drive

A common magnetic storage device that reads and writes data on metal disks inside a sealed case.

hardware

The physical components of a computer system.

home page

The starting page on a Website.

IBM

Abbreviation for the computer company called International Business Machines Corporation.

icon

A small picture used in a WIMP system to represent a program, folder or file.

index

A set of links that can be used to locate records in a data file.

Information Services Manager

The head of the computer department.

information technology

The study and practice of techniques or use of equipment for dealing with information.

input

Data put into a system / to put data into a system.

input device

A piece of equipment used for entering data or controlling a computer.

insertion point

The position where something is put into a file.

Internet, (the)

The connection of computer networks across the world.

jam

To get stuck in one position.

justify

To insert spaces so that lines of a text are aligned on both the left and right sides at the same time.

KB

Abbreviation for a kilobyte.

keyboard

The main electronic input device that has keys arranged in a similar layout to a typewriter.

keypad

A small keyboard with a few keys used for a special purpose.

Kilobyte

A capacity of 2^{10} bytes, i.e. 1024 bytes.

LAN

Acronym for local area network.

laptop (computer)

The largest type of portable computer.

load module

The program which is directly executable by the computer.

local area network

Computers connected together over a small distance.

machine code

A computer language that consists entirely of a combination of 1s and 0s.

main memory

The electronic memory that holds the programs and data being used.

mainframe (computer)

The largest and most powerful type of computer. It is operated by a team of professionals.

Megabyte

A unit of capacity equal to 2^{20} bytes, i.e. 1024 kilobytes.

Megahertz

A unit of frequency equal to 1 million cycles per second.

memory (store)

The part of a computer system that is used for storing programs and data.

menu

A list of options displayed on a computer screen.

menu bar

A row of icons on a display screen that open up menus when selected.

mesh topology

An arrangement of computers in a network where every computer is connected to every other computer by a separate cable.

microchip

An electronic integrated circuit in a small package.

microcomputer

A personal computer, smaller and less powerful than a mainframe or a minicomputer.

modem

An electronic device for converting signals to enable a computer to be connected to an ordinary telephone line. The term comes from an abbreviation of MODulator / DEModulator.

monitor

The main output device used to display the output from a computer on a screen. See **VDU**.

mouse

A common cursor control input device used with a graphical user interface. It has two or three button switches on top and a ball underneath that is rolled on a flat surface.

mouse button

A switch on a mouse that is pressed to select an object on the screen.

multimedia

The combination of text, graphics, animation, sound, and video.

multimedia computer

A computer suitable for running multimedia programs. It usually has a sound card and a CD-ROM drive.

Net, (the)

The common name for the Internet.

network

A combination of a number of computers and peripheral devices connected together / to connected a number of computers and peripheral devices together.

network-compatible

Describing software that can be run on a network with shared files rather than as a stand alone piece of PC software.

operating system

The set of programs that control the basic functions of a computer.

output

Data brought out of a system / to bring data out of a system.

output device

A piece of equipment used to bring data out of a system.

package

An application program or collection of programs that can be used in different ways.

PASCAL

A programming language designed as a tool to assist the teaching of programming as a systematic discipline.

password

A method of security in which the user has to enter a unique character string before gaining access to a computer system.

PC

Personal computer.

PIN

Abbreviation for personal identification number.

PL/I

Programming language I. A programming language developed by the US IBM user's group, implementing the best features of COBOL, FORTRAN and ALGOL.

portable (computer)

A computer that is small and light enough to be carried from place to place. It can usually be powered by batteries.

printer

A common output device used for printing the output of a computer on paper.

procedure

A subsection of a high-level program designed to perform a particular function.

process

To manipulate the data according to the program instructions.

processor

The part of a computer that processes the data.

program

A set of instructions written in a computer language that control the behaviour of a computer / to write a set of instructions for controlling a computer using a computer language.

programmer

A person who writes computer programs.

programming

The processes of writing a computer program using a computer language.

RAM

Acronym for random access memory – memory that can be read and written to by the processor.

refresh rate

The frequency at which the image is re-drawn on a display screen.

register

A small unit that is used to store a single piece of data or instruction temporarily that is immediately required by the processor.

ring network

A network constructed as a loop of unidirectional links between nodes.

ring topology

A physical layout of a network where all the computers are connected in a closed loop.

ROM

Acronym for read-only memory.

RS/6000

A model of IBM computer which is UNIX based.

ruler

A horizontal line containing markings indicating measurements on the display screen.

run

To execute a program, i.e. to get a program to process the data.

save

To copy a program or data to a storage device.

scan

To copy using a scanner.

screen (display)

The front surface of a computer monitor where the output of a computer is displayed.

search engine

A program designed to find information on the World Wide Web according to data entered by the user. Search engines are usually accessed from special websites.

secondary storage

Memory used for storing data that is not currently being used.

server

A main computer that provides a service on a network.

software

The programs and data used in a computer.

spreadsheet

A program that manipulates tables consisting of rows and columns of

cells and displays them on a screen. The value in a numerical cell is either typed in or is calculated from values in other cells. Each time the value of a cell is changed the values of dependent cells are recalculated.

standard

A publicly available definition of a hardware or software component resulting from national, international, or industry agreement.

star network

A simple network topology with all links connected directly to a single central node.

star topology

A physical layout of a network where all the computers are connected by separate cables to a central hub.

status bar

A narrow band across the bottom of the screen that displays useful information for the user.

storage device

A piece of equipment used for reading from and writing to a storage medium.

stylus

An electronic I/O device that is used to draw or write on the screen.

subscriber

A user who becomes a member of a newsgroup.

system board

The main circuit board of a computer containing the micro-processor chip. Other devices will be attached to this board.

systems routine

Utility programs provided by the computer operating system. These might be used for converting numerical data into different formats, or performing operations on dates.

terminal

A network device used to input and output data (usually a basic computer).

title bar

A narrow strip across the top of a window in a WIMP system that indicates what is inside the window.

toolbar

A row of icons displayed on a screen that start common program functions when clicked with a mouse.

toolbox

A set of icons displayed on a screen for selecting common program editing functions. For example, a graphics package usually has a toolbox containing icons for choosing the line width, the line colour, for creating different common shapes, and for rotating images.

topology

The physical layout of a network.

undo

To restore a file to the condition it was in before the last change was made.

upgrade

To add component to improve the features or performance of a system.

upgradeable

Designed so that components can be added to improve the features or performance of the system.

user

An individual or group making use of the output of a computer system.

VDU

Abbreviation for visual display unit / another name for a computer monitor.

vertical refresh rate

The number of times per second that an image is written on a TV or

computer screen, measured in kilohertz.

virtual reality

A simulated three dimensional environment that surrounds the user and is generated by a computer.

virus

A program written deliberately to damage data or cause a computer to behave in an unusual way.

WAN

Acronym for wide area network.

war game

A computer game which emulates warfare.

webpage

A hyperlinked page in a web network system.

website

A set of pages on the World Wide Web.

word processing

The process of typing and editing text using a word processor.

word processor

A type of computer application program used for typing and editing text documents.

workstation

A desk area used for working with a computer system.

World Wide Web, (the)

An information service on the Internet that allows documents pages to be accessed using hyperlinks.

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