Optimization

1) Absolute extrema:

Absolute Maxima and Minima of function:

Let f be a defined on an interval D that contains the number c. Then:

$f(c)$ is the *absolute maximum* of *f* on *D* if $f(x) \le f(c)$ for all *x* in *D*

$f(c)$ is the *absolute minimum* of *f* on *D* if $f(x) \ge f(c)$ for all *x* in *D*

Collectively, absolute maxima and minima are called absolute extrema.

Optimization

2) The Second Dervative Test for Absolute Extrema:

Suppose that $f(x)$ is continuous on an interval D where $x = c$ is only critical number and that $f'(c) = 0$. Then,

If $f''(c) > 0$, the *absolute minimum* of f on D is $f(c)$

If $f''(c) < 0$, the *absolute maximum* of f on D is $f(c)$

Example 1:

A manufacturer estimates that when q units of a particular commodity are produced each month, the total cost will be

$$C(q) = q^2 + 8q + 20 \quad \textbf{dollars}$$

and all *q* units can be sold at a price of

$$p(q) = 2(40 - q) \quad \textbf{dollars per unit}$$

Determine the level of production that results in maximum profit. What is the maximum profit?

Solution

The revenue is

$$R(q) = q.p(q) = 2q(40 - q)] = 80q - 2q^2$$

The profit is

$$P(q) = R(q) - C(q) = 80q - 2q^2 - (q^2 + 8q + 20)$$

$$= -3q^2 + 72q - 20$$

The derivative of P: $P'(q) = -6q + 72$

$$P'(q) = 0 \Leftrightarrow q = 12$$

$$P''(q) = -6 < 0$$

So maximum profit occurts when q= 12 units are produced.

The maximum profit is 1276 dollars.

The graph of the profit function is shown in Figure 4.

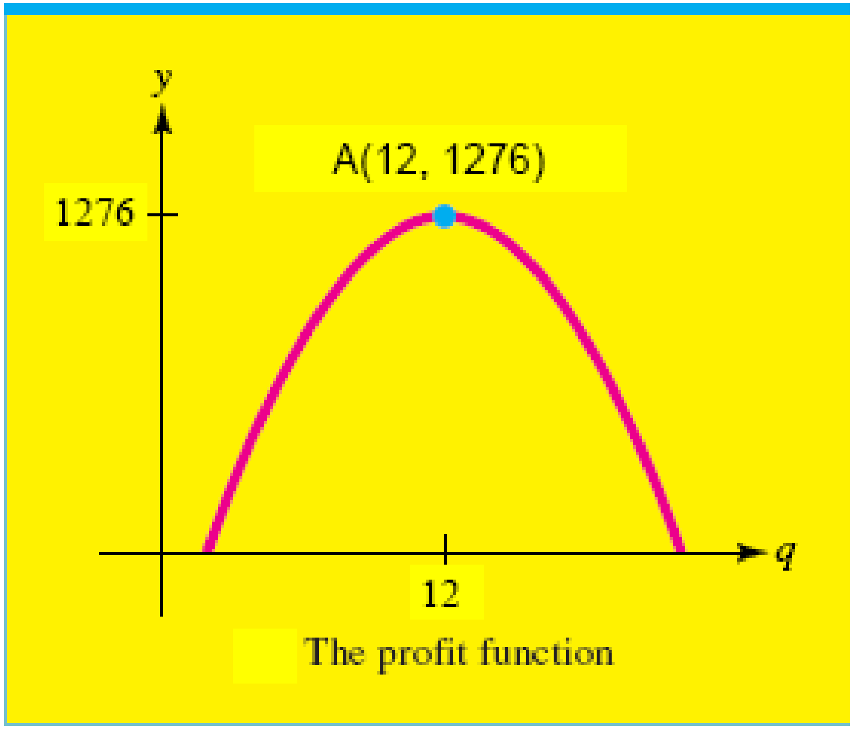

Figure 4. Graphs of profit. $y = -3q^2 + 72q - 20$

Example 2:

A bookstore can obtain a certain gift book from the publisher at a cost of $3 per book. The bookstore has been offering the book at a price of $15 per copy and, at this price, has been selling 200 copies a month. The bookstore is planning to lower its price to stimulate sales and estimates that for each $1 reduction in the price, 20 more books will be sold each month. At what price shoud the bookstore sell the book to generate the greatest possible profit?

Thank you for listening