

NET EXCESS PROFIT

In section, you will discover that this procedure applies to a variety of practical situations in business, economics and life, social sciences.

Suppose that t years from now, two investment plans will be generating profit $P_1(t)$ and $P_2(t)$, respectively, and that the respective rates of profitability, $P'_1(t)$ and $P'_2(t)$, satisfy $P'_2(t) \geq P'_1(t)$ for the first N years ($0 \leq t \leq N$). Then

$$E(t) = P_2(t) - P_1(t)$$

represents the **excess profit** of plan 2 over plan 1 at time t and the **net excess profit** $NE = E(N) - E(0)$ over the time period $0 \leq t \leq N$ is given by the definite integral

$$\begin{aligned} NE &= E(N) - E(0) = \int_0^N E'(t) dt \\ &= \int_0^N [P'_2(t) - P'_1(t)] dt \end{aligned}$$

which can be interpreted geometrically as the area between the curves $y = P'_1(t)$ and $y = P'_2(t)$ (see Figure 6.10). Here is an example.

Suppose that t years for now, one investment will be generating profit at the rate of $P'(t) = 50 + t^2$ hundred dollars per years, while a second investment will be generating profit at the rate of $P'(t) = 200 + 5t$ hundred dollars per years.

- For how many years does the rate of profitability of the second investment exceeded that of the first?
- Compute the net excess profit for the time period determined in part (a) interpret the net excess profit as an area.

Solution

(a) The rate of profitability of the second investment exceeds that of the first until

$$\begin{aligned}P'_1(t) &= P'_2(t) \\50 + t^2 &= 200 + 5t \\t^2 - 5t - 150 &= 0 \\(t - 15)(t + 10) &= 0 \\t &= 15 \text{ years (reject } t = -10)\end{aligned}$$

(b) The net excess profit for the time period $0 \leq t \leq 15$ is given by the definite integral

$$\begin{aligned}\text{NE} &= \int_0^{15} [P'_2(t) - P'_1(t)] dt = \int_0^{15} [(200 + 5t) - (50 + t^2)] dt \\&= \int_0^{15} (150 + 5t - t^2) dt \\&= \left(150t + \frac{5}{2}t^2 - \frac{1}{3}t^3 \right) \Big|_0^{15} = 1,687.50 \text{ hundred dollars}\end{aligned}$$

that is, \$168,750.

The rate of profit curves for the two investments are shown in Figure 6.11. The net excess profit is the area of the (shaded) region between the curves.