Vector Functions and Space Curves

In this section, we will learn about:

Vector functions and drawing their corresponding space curve

In general, a function is a rule that assigns to each element in the domain an element in the range.

A vector-valued function, or vector function, is simply a function whose:

- Domain is a set of real numbers.
- Range is a set of vectors.

We are most interested in vector functions **r** whose values are three-dimensional (3-D) vectors.

- This means that, for every number t in the domain of **r**, there is a unique vector in V_3 denoted by $\mathbf{r}(t)$.

If $f(t)$, $g(t)$, and $h(t)$ are the components of the vector $\mathbf{r}(t)$, then f, g, and h are real-valued functions called the component functions of **r.**

We can write:

$$\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle = f(t)\,\mathbf{i} + g(t)\,\mathbf{j} + h(t)\,\mathbf{k}$$

We usually use the letter t to denote the independent variable because it represents time in most applications of vector functions.

If $\mathbf{r}(t) = \langle t^3, \ln(3-t), \sqrt{t}\rangle$ then the component functions are:

$$f(t) = t^3 \qquad g(t) = \ln(3-t) \qquad h(t) = \sqrt{t}$$

By our usual convention, the domain of **r** consists of all values of *t* for which the expression for **r**(*t*) is defined.

- The expressions t^3, ln(3 – *t*), and are all defined when 3 – *t* > 0 and *t* ≥ 0.
- Therefore, the domain of **r** is the interval [0, 3).

The limit of a vector function **r** is defined by taking the limits of its component functions as follows.

If **r**(*t*) = ‹*f*(*t*), *g*(*t*), *h*(*t*)›, then $\lim_{t\to a}\mathbf{r}(t) = < \lim_{t\to a} f(t), \lim_{t\to a} g(t), \lim_{t\to a} h(t)\rangle$ provided the limits of the component functions exist.

If $\lim_{t\to a}\mathbf{r}(t) = L$, this definition is equivalent to saying that the length and direction of the vector **r**(*t*) approach the length and direction of the vector **L**.

Find $\lim_{t\to o}\mathbf{r}(t)$ where $\mathbf{r}(t) = (1+t^3)\mathbf{i} + te^{-t}\mathbf{j} + \frac{\sin t}{t}\mathbf{k}$

According to Definition 1, the limit of **r** is the vector whose components are the limits of the component functions of **r**:

$$\lim_{t\to 0}\mathbf{r}(t) = \left[\lim_{t\to 0}(1+t^3)\right]\mathbf{i} + \left[\lim_{t\to 0} te^{-1}\right]\mathbf{j} + \left[\lim_{t\to 0}\frac{\sin t}{t}\right]\mathbf{k}$$

$$= \mathbf{i} + \mathbf{k}$$