Duy Tân University

Natural Sciences Department

Module 1:

Series

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1. Definition

-Let $\{a_n\}$ be an infinite sequence.

Then, $a_1 + a_2 + a_3 + ... + a_n + ... = \sum_{n=1}^{\infty} a_n$

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Let:

$$s_1 = a_1$$

$$s_2 = a_1 + a_2$$

$$s_3 = a_1 + a_2 + a_3$$

$$s_n = a_1 + a_2 + a_3 + ... + a_n$$

1. Definition

If the sequence s_n is convergent and $\lim s_n = s$ exists as a real number, then the series is called convergent and we write

$$\sum_{n=1}^{\infty} a_n = s$$

The number s is called the sum of the series.

Otherwise, the series is called divergent.

Example:

Are the following series convergent or divergent?

$$a.\ \sum_{n=1}^{\infty} n$$

$$b.\ \sum_{n=1}^{\infty} \left(\frac{4}{7}\right)^n$$

$$c.\ \sum_{n=1}^{\infty} (-1)^n$$

Steps to determine the convergence or divergence of series:

- Determine a_n
- Caculate s_n
- Find $\lim s_n$
  - \+ $\lim s_n = s \in \mathbb{R}$: $\sum_{n=1}^{\infty} a_n$: convergent and $\sum_{n=1}^{\infty} a_n = s$
  - \+ $\lim s_n = \infty$
  - \+ Don't exist the limit of s_n

  (for the last two cases) $\sum_{n=1}^{\infty} a_n$: divergent

Special series:

$$\sum_{n=1}^{\infty} q^n$$: geometric series.

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$: p- series.

2.Some test:

-The Comparison test.

- The Integral test.

- The Limit Comparison test.

- The Alternating Series test.

- The Ration test.

- The Root test.

2.Some test:

a. The Test for Divergence.

- Let $\sum_{n=1}^{\infty} a_n$.

If $\lim a_n \neq 0$ or $\lim a_n$ does not exist then the series

$\sum_{n=1}^{\infty} a_n$ is divergent

2.Some test:

b. The Comparison test.

- Let $\sum_{n=1}^{\infty} a_n ; \sum_{n=1}^{\infty} b_n$ are positive series and if

$$\lim \frac{a_n}{b_n} \in (0; +\infty)$$

Then, either both $\sum_{n=1}^{\infty} a_n ; \sum_{n=1}^{\infty} b_n$ convergent or both divergent.